For some questions you may leave answers as **ununsimplified numerical expressions**. The expressions could include mathematical notation such as $\frac{1}{3}\sqrt{\text{ln}(3.72)}$, $\log_{10}(45)$, $10^{2.07}$ etc. For example

$$\bar{y} = \frac{1.2 + 1.3 + 1.8}{3} \quad \text{and} \quad s = \sqrt{\frac{(1.2 - \bar{y})^2 + (1.3 - \bar{y})^2 + (1.8 - \bar{y})^2}{2}}$$

would be fine unsimplified numerical expressions. The second expression is fine because $\bar{y}$ had been given a numerical expression already. **In these cases you do not need to evaluate or simplify the expression.**

(1) (8 points) A type plywood is made of 3 layers. The thicknesses of the layers are independent random variables with means and standard deviations as follows. Give **unsimplified numerical expressions** for the mean and standard deviation of the total thickness.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.094</td>
<td>0.001</td>
</tr>
<tr>
<td>2</td>
<td>0.156</td>
<td>0.002</td>
</tr>
<tr>
<td>3</td>
<td>0.234</td>
<td>0.003</td>
</tr>
</tbody>
</table>

(2) (8 points) If a dimension of a mechanical part is normally distributed, how small must the standard deviation be if 95% of such parts are to be within specifications of 2 cm $\pm$ 0.002 cm when the mean dimension is 2 cm?
(3) (24 points) The resistance of a particular type of resistor is normally distributed with mean \( \mu = 9.9 \, \Omega \) and standard deviation \( 0.1 \, \Omega \).

(a) If a single resistor is chosen randomly, what is the probability that the resistor has a resistance of greater than 10 \( \Omega \)?

(b) If 4 resistors are chosen randomly, give an unsimplified numerical expression for the probability that exactly 2 of the 4 resistors have a resistance greater than 10 \( \Omega \).

(c) If 4 resistors are chosen randomly, what is the probability that the mean (average) resistance of the 4 resistors, \( \bar{X} \), exceeds 10 \( \Omega \)?
(4) (13 points) In a study report in 2006 in the *Canadian Journal of Civil Engineering*, concrete compressive strengths for a particular type of concrete were found to be modeled well as Weibull with $\alpha \approx 34 \text{ MPa}$ and $\beta \approx 8$.

(a) Give an unsimplified numerical expression for the probability that the compressive strength of a concrete sample is less than 25 MPa.

(b) Find the 5th percentile, the 0.05 quantile, of compressive strengths for this type of concrete.

(5) (14 points) For predicting wear life (Y) of a film lubricant from speed (X1) and load (X2), we find the fitted equation $\ln (Y) \approx 5 - 0.2 \ln \text{(speed)} - 0.4 \ln \text{(load)}$.

(a) Write the corresponding fitted equation for $Y$ in simplest form.

$Y \approx$

(b) What is the effect on wear life associated with a doubling of the speed? Give me a numerical answer, not just "increase" or "decrease".
(6) (12 points) Random variables X and Y are combined into an output result defined by

\[ U = g(X, Y) = \frac{X + 5}{Y} \]

For the random variables X and Y

\[
\begin{align*}
E(X) = \mu_X &= 10 & \text{Var}(X) = \sigma_X^2 &= 1.5 \\
E(Y) = \mu_Y &= 1 & \text{Var}(Y) = \sigma_Y^2 &= 0.2
\end{align*}
\]

(a) Using propagation of error, find the approximate variance for \( g(X,Y) \).

(b) Which random variable, X or Y, contributes most to the variability of the output result \( g(X,Y) \)? How did you decide?
(7) (21 points) Suppose the number of major bumps in a newly constructed road is 1.5 bumps per mile. Let X be the number of bumps in 3 miles of road.

(a) What is a possible distribution for X? Choose from binomial, geometric, Poisson, normal, exponential, or Weibull.

(b) If we check 3 miles of road where on average there are on average 1.5 bumps per mile, give an unsimplified numerical expression for the probability that we find no bumps.

(c) How many miles of road would we need to check to have a 95% chance of finding at least one bump if on average there are 1.5 bumps per mile?