(1) (a)

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>2</td>
<td>1155.15</td>
<td>577.575</td>
<td>5.7</td>
</tr>
<tr>
<td>Error</td>
<td>103</td>
<td>10435.42</td>
<td>101.315</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>105</td>
<td>11590.57</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\bar{Y}_i = \frac{\sum n_i \bar{Y}_i}{n_i} = \frac{63 \times 55.1 + 22 \times 47.6 + 21 \times 49.4}{106} = 52.4
\]

\[
SS_{\text{groups}} = \sum n_i (\bar{Y}_i - \bar{Y}_.)^2 = 1155.15
\]

\[
SS_{\text{Error}} = \sum (n_i - 1)s_i^2 = 10435.42
\]

b) Two contrasts are orthogonal \( \iff \sum \frac{c_i d_i}{n_i} = 0 \)

\[
\begin{align*}
n_1 &= 63 \\
n_2 &= 22 \quad \frac{0 \times (-2)}{63} + \frac{1 \times (1)}{22} + \frac{(-1) \times 1}{21} = -\frac{1}{462} \neq 0
\end{align*}
\]

These 2 contrasts are not orthogonal, so their SS do not add up to the SS for testing if both contrasts are true. Both of these contrasts being true would be equivalent to all three population means being equal.

(2) (a) \[
\frac{(1.645 + 0.84)^2 \times 2 \times 12^2}{4^2} = 112 \text{ boards}
\]

b) \[
P\left(Z > 1.645 - \frac{4}{\sqrt{2 \times 12^2}}\right) = P(Z > -0.46) = 0.68
\]

c) Decrease \( \sigma \) if possible by pairing or by using more consistent material.

(3)

(a) \[
\bar{y}_w = \frac{6(9) + 3(10) + 18(15)}{27} = 13.1
\]

b) \[
S_w^2 = \frac{5(4)^2 + 2(5)^2 + 17(5)^2}{24} = 23.125
\]
c) \( SS_{\text{corr}} = \sum n_i (\bar{y}_i - \bar{y})^2 \)

d) 

<table>
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<th>F</th>
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<tbody>
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<td>Species</td>
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<td>97.3</td>
<td>4.2</td>
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<tr>
<td>Error</td>
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<td>55</td>
<td>23.1</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>26</td>
<td>749</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

e) \( 0.01 < p < 0.05 \)

(4) 

a) 

\[ df_{\text{model}} = 4 \quad SS_{\text{model}} = 4(21.5 - 17.1)^2 + \ldots + 4(17.75 - 17.1)^2 = 226 \]

\[ df_{\text{error}} = 5 \quad SS_{\text{error}} = 3(1.29^2) + \ldots + 3(1.70^2) = 5.29 \]

\[ F = \frac{MS_{\text{model}}}{MS_{\text{error}}} = \frac{56.525}{5.29} = 10.7 \quad \text{Root MSE} = \sqrt{5.29} = 2.3 \]

\[ 2 \text{ vs } 1 \quad \text{Est} = 5.75 \quad T = \frac{5.75}{\sqrt{5.29(2/4)}} = 3.53 \quad \text{SE} = \sqrt{5.29(2/4)} = 1.628 \]

b) \( 10.7 > 4.89 \)   Yes, significant at \( \alpha = 0.01 \)

c) Dunnett = 2.36

\[ 2 \text{ vs } 1 \quad t = 3.53 \]

\[ 3 \text{ vs } 1 \quad t = 6.14 \quad \text{are significantly different from 1} \]

(5) 

\[ S_{W} = \sqrt{\frac{34.38}{16}} \]

(6) 

\[ S_{W}^2 = \frac{11(1.79)^2 + 11(1.76)^2 + 7(1.34)^2}{11 + 11 + 7} = 2.82 \]

a) 

\[ S_{W} = \sqrt{2.82} = 1.68 \]

This looks reasonable.
b) 
\[ SS_{\text{model}} = \sum n_i (\bar{y}_i - \bar{y})^2 \]
\[ \bar{y} = \frac{12(8.08) + 12(7.86) + 8(6.47)}{12 + 12 + 8} = 7.595 \]
\[ SS_{\text{model}} = 12(8.08 - 7.595)^2 + 12(7.86 - 7.595)^2 + 8(6.47 - 7.595)^2 = 13.79 \]

c) \[ F = \frac{13.79/2}{2.82} = 2.44 \]

Using this test at the \( \alpha = 0.05 \) level, we do not have sufficient evidence to conclude that the water salinity affects growth of trout. We cannot conclude “water salinity has no effect on growth of trout.” Why not?

d) \( H_0: \frac{1}{2} \mu_1 + \frac{1}{2} \mu_2 - \mu_3 = 0 \)
\[ \text{contrast} = \frac{8.08 + 7.86}{2} - 6.47 = 7.97 - 6.47 = 1.5 \]
\[ SE(\text{contrast}) = 1.68 \sqrt{\left(\frac{1}{12}\right)^2 + \left(\frac{1}{12}\right)^2 + \left(\frac{1}{8}\right)} = 0.69 \]

Since the sample sizes are the same in the groups being averaged, the average of the averages is the same as the average of the 12 + 12 = 24 fish, so \( SE(\text{contrast}) = 1.68 \sqrt{\frac{1}{24} + \frac{1}{8}} = 0.69 \) also,
\[ t = \frac{\text{contrast}}{SE(\text{contrast})} = \frac{1.5}{0.69} = 2.19 \]

At the \( \alpha = 0.05 \) level, we reject \( H_0: \mu_{\text{sea}} = \mu_{\text{others}} \). Notice that this seems inconsistent with part c): Part c), water salinity has no effect, and part d), sea water is different from others. This is one of the drawbacks of viewing these reject/accept decisions as black and white yes/no answers.
(7) (a)  

\[ S_w^2 = \frac{6(.09)^2 + 7(.18)^2 + 7(.15)^2 + 7(.17)^2}{6 + 7 + 7 + 7} = \frac{.6352}{27} = 0.0235 \]

\[ \bar{y}_c = \frac{7(1.64) + 8(1.53) + 8(1.01) + 8(0.93)}{7 + 8 + 8 + 8} = \frac{39.24}{31} = 1.269 \]

\[ MS_{Treat} = \frac{7(1.64 - 1.269)^2 + 8(1.53 - 1.269)^2 + 8(1.01 - 1.269)^2 + 8(0.93 - 1.269)^2}{4 - 1} = 0.988 \]

\[ F = \frac{MS_{Treat}}{\sigma^2} = \frac{0.988}{0.0235} = 42.04 \quad F_{0.05,3,27} = 2.96 \]

42.04 > 2.96  Reject \( H_0 \) at the 0.05 level.

b)  

\[ SE = \sqrt{0.0235 \cdot \sqrt{\frac{1}{4} \times \frac{1}{7} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}}} = 0.0552 \]

\[ t = \frac{1}{2}(1.64) + \frac{1}{2}(1.01) - \frac{1}{2}(1.53) - \frac{1}{2}(0.93) - 0 = \frac{0.095}{0.0552} = 1.721 \]

\[ t_{0.05,27} = 2.052 \quad 1.721 < 2.052 \quad \text{Do not reject } H_0 \text{ at the 0.05 level} \]

c)  

\[ t_{0.01,27} = 2.771 \]

\[ SE = 0.0552 \]

\[ 0.095 \pm 2.771(0.0552) \]

\[ 0.095 \pm 0.153 \]

\[ (-0.058, 0.248) \]

d) want \( \frac{z_{0.95} \cdot SE}{\sqrt{n}} \leq 0.1 \)

\[ 1.645 \cdot \frac{0.0552}{\sqrt{n}} \leq 0.1 \]

\[ n = \left[ \frac{1.645(0.1533)}{0.1} \right]^2 \approx 7 \]

e) \[ SE\left( \frac{\hat{\mu}_1 + \hat{\mu}_3 - \hat{\mu}_2 + \hat{\mu}_4}{2} \right) = \sigma \sqrt{\frac{1}{4} \times \frac{1}{n} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{n}} = \sigma \sqrt{\frac{1}{n}} \]

For \( n = 25 \) \( SE \approx 0.1533/5 \approx 0.03 \)

\[ P\left( Z > 1.645 - \frac{0.1}{0.03} \right) = P(Z > -1.69) = 0.9545 \]
(f) We need $1.645 - \frac{0.1}{0.1533\sqrt{n}} = -2.326$

Solving for $n$, we find, $n \approx 38$ animals (rounded up)