1. Obtain the matrix representations of the spin operators $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$ starting from $\hat{S}_+ \chi_+ = 0$, $\hat{S}_- \chi_- = 0$, $\hat{S}_+ \chi_- = \hbar \chi_+$, and $\hat{S}_- \chi_+ = \hbar \chi_-$. 

2. Obtain the eigenspinors of $\hat{S}_y$, $\chi^{(y)}_+$ and $\chi^{(y)}_-$, and show that

$$
\begin{pmatrix}
a \\
b
\end{pmatrix} = \frac{1}{\sqrt{2}} \left[ (a - ib)\chi^{(y)}_+ + (a + ib)\chi^{(y)}_- \right]
$$

3. An electron is known to be in the spin state $\chi_+$. Show that in this state $\langle \hat{S}_x \rangle = \langle \hat{S}_y \rangle = 0$. Explain this result geometrically.

4. Show that it is impossible for a spin-1/2 particle to be in a state $\chi = \begin{pmatrix} a \\ b \end{pmatrix}$ such that $\langle \hat{S}_x \rangle = \langle \hat{S}_y \rangle = \langle \hat{S}_z \rangle = 0$.

5. Obtain (using the lowering-operator technique) the Clebsch-Gordon coefficients associated with the coupling of the orbital and spin angular momentum of a P electron.