PHYS 2021 – Final Exam

Friday, May 17, 2013

Time: 120 min

Name: ____________________________________

Instructions

- This is a closed book test. A letter-size formulae sheet and scratch paper are allowed. A calculator is allowed.

Good luck!

Physical constants:

Speed of light: c = 3.0 x 10^8 m/s
Stephan-Boltzmann constant: σ = 5.6703x10^{-8} W/m^2K^4
Boltzmann constant: k_B = 1.38065x10^{-23} J/K
Wien's displacement law constant: b = 2.898x10^{-3} m·K
Planck's constant: h = 6.626 x 10^{-34} J·s = 4.135667 x 10^{-15} eV·s
$\hbar$ = 1.054572 x 10^{-34} J·s = 6.582 x 10^{-16} eV·s
Mass of electron: m_e = 9.109 x 10^{-31} kg = 510.0 keV / c^2
Mass of proton: m_p = 1.67 x 10^{-27} kg = 938.3 MeV / c^2
Unified mass unit: u = 1.66 x 10^{-27} kg = 931.5 MeV / c^2
Electron charge: e = 1.602x10^{-19} C
Coulomb force constant: k_e = 8.988x10^9 N·m^2/C^2
1 eV = 1.60 x 10^{-19} J
Ground state of hydrogen atom: E_1 = *mke2e42ħ2*≈13.6 eV
Ground state of infinite square well: E_1 = *ħ2π22mL2*
Integrals: −∞*e−x2/a2dx=πa*

x2ebxdx=(x2b−2xb2+2b3)ebx

x3ebxdx=(x3b−3x2b2+6xb3−6b4)ebx

1. (*6 points*) You are an observer at an outpost at the far end of the galaxy. You see a spaceship go by and measure its length to be 85 meters. Your catalogue lists the ship as being 100 m long.
   a. (*2 points*) What is the speed of the ship?

   b. (*2 points*) You continue observing the ship and see that it takes 1 year to reach its destination. How much time passes on the ship captain's clock?

   c. (*2 points*) Indicate the two measured time periods (yours and captain's) on a space-time diagram.

2. (6 *points*) In a laboratory frame of reference *S*, an electron of rest mass m_0=0.5 MeV travelling with speed *v*=0.6*c* meets a stationary positron (also m_0=0.5 MeV) and they briefly form an electron-positron pair.

   a. (2 *points*) What is the momentum of the pair in *S*?

   b. (2 *points*) The particles then annihilate, producing two photons. How much energy is released as electromagnetic waves?

   c. (2 *points*) Would you expect the produced photons to have the same frequencies? If not, which one would have a higher frequency? Explain.

3. (*8 points*) For a hydrogen atom:
    a. (*2 points*) Find the wavelength of the photon emitted in a transition from n=2 to n=1.

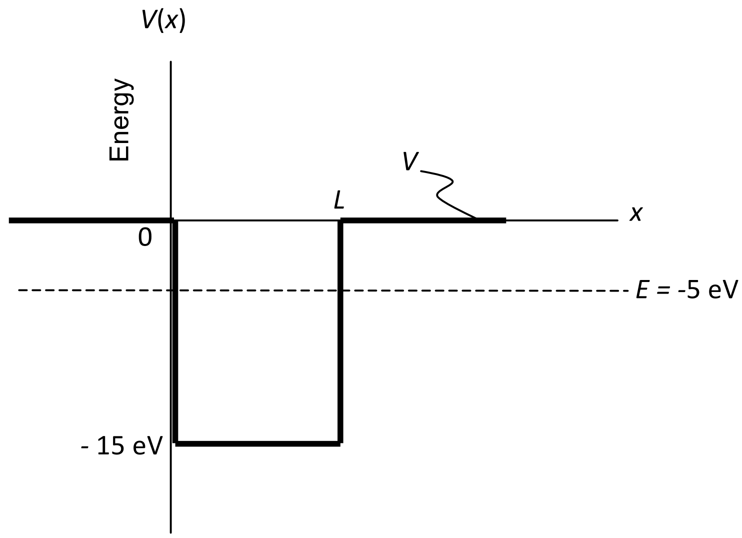
    b. (*2 points*) For a quantum number n=2, enumerate the distinct states that the electron can be in. What is the total number of such states?

    c. (*1 point*) If you measure the *z*-component of the electron's orbital angular momentum, L_z, in a state with (n=4, l=3), what values can you find?

    d. (*3 points*) Using classical physics and a Bohr model, write the expressions for the electron's kinetic energy and potential energy. Using these expressions, derive the expression for the radius of the first Bohr orbit.

4. (*12 points*) An electron is in the n=3 state with energy E inside a potential well of width L shown below.

a. (*2 points*) Sketch a possible wavefunction of this electron.



b. (*2 points*) What is the uncertainty in its velocity?

c. (*2 points*) What is the expectation value of its momentum?

d. (*2 points*) Estimate the probability of the electron being in a small region of space $0.495\ L < x < 0.505\ L$.

e. (*2 points*) The electron is excited from the shown state (E) by a photon of momentum $p = 6$ eV/c. Sketch the new wavefunction of the electron.

f. (*2 points*) What is the speed of this excited electron at $x=L/2$?

5. (*8 points*) The electron in the ground state (n=1, l=0) of hydrogen atom is characterized by the normalized radial function

$R10=2a03e-r/a0$, where a_0 is the Bohr radius, and r is the distance from nucleus.

a) *(4 points*) Find the distance r where the electron is ***most likely*** to be found (i.e. the most probable value of r).

b) (*4 points*) Find the *expectation value* of r (i.e. the average distance of the electron from the nucleus).

6. (*3 points*) Name one atom for which the Stern-Gerlach experiment (where an atomic beam is passed through an inhomogeneous magnetic field and detected on a collector plate) would produce a single line, and one atom for which the experiment would produce two lines. Explain. Consider all atoms in ground state.