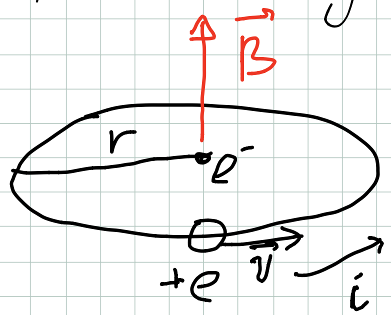
#5. Magnetic field inside atom



This is what the electron "sees": a current loop.

$$i = \frac{dq}{dt} = \frac{e}{T} = \frac{ev}{2\pi r}$$

$$B = \frac{\mu_0 i}{2r} = \frac{\mu_0 e v}{4\pi r^2}$$, where v is e^- "velocity"

For example, from Bohr quantization:

$$L = rmv = n\hbar \Rightarrow v = \frac{\hbar}{m_e r} \quad (n=1)$$

$$B = \frac{\mu_0 e\hbar}{4\pi m_e r^3} = \qquad r \approx a_0$$

$$= \frac{4\pi * 10^{-7} \cdot 1.6 \cdot 10^{-19} \cdot 1.05 \cdot 10^{-34}}{4\pi\ 9.1 \cdot 10^{-31} \cdot (0.5 \cdot 10^{-10})^3} =$$

$$= 1.5 \cdot 10^1 = 15 \text{ Tesla}$$

It is on the same order as what one obtains in problem 7-40 from fine-splitting

#6 $\quad L_z = m\hbar \qquad \vec{L} = \vec{r} \times \vec{p}$

a) $\hat{L} = \hat{r} \times \hat{p} = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ p_x & p_y & p_z \end{vmatrix}$

$$\hat{L}_z = \begin{vmatrix} r_x & r_y \\ p_x & p_y \end{vmatrix} \vec{k} = r_x \hat{p}_y - r_y \hat{p}_x =$$

$$= -i\hbar \left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x} \right)$$

b) $\hat{L}_z = -i\hbar \frac{\partial}{\partial \varphi}$

c) *Eigenvalue equation:* $\hat{L}_z \psi = m\hbar \psi$

$$\psi = R_{n\ell}(r) Y_{\ell m}(\theta, \varphi) = R(r) f(\theta) g(\varphi)$$

Thus, $\hat{L}_z g = m\hbar g$ as $\hat{L}_z$ acts only on φ

But $g_m = e^{im\varphi}$, so

$$\hat{L}_z g = -i\hbar \frac{\partial}{\partial \varphi} e^{im\varphi} = -i\hbar \cdot im\, e^{im\varphi} = m\hbar g.$$