RLC Frequency Response

1. Introduction

The student will analyze the frequency response of an RLC circuit excited by a sinusoid. Amplitude and phase shift of circuit components will be analyzed at different frequencies through theory, simulation and experimental results. Circuit current and impedances will be calculated using these measurements along with voltage phasors. The student will be introduced to complex voltages and impedances along with phasor notation.

2. Background

2.1. Complex Electrical Notation

Current, voltage and impedance in an electrical system can contain both real and imaginary components; the imaginary components are a result of inductors and capacitors. While a discussion of the mechanism behind these imaginary properties will not be presented, the mathematics that represent complex electrical notation will be described.

Complex notation of an electrical property (current, voltage or impedance) is described in Equation (1) where X represents the real component, Y represents the imaginary component, and Z represents the complex combination of these two.

$$Z = X + jY \qquad j = \sqrt{-1} \tag{1}$$

Complex electrical notation can also be written in polar form that describes the electrical property's magnitude at some angle.

As an example, treat the illustration in Figure 1 as that describing the current through an arbitrary electrical component. The current would have a real component of 3, an imaginary component of 4 and a magnitude of 5. Thus the current could be written as I = $3 + j4$ or $5\angle 53.1°$ Amps.

To clarify, the real component of an electrical property is typically the only one that can be directly measured in a laboratory setting. Using an ammeter to measure the current in the described example would provide an answer of 3 Amps.

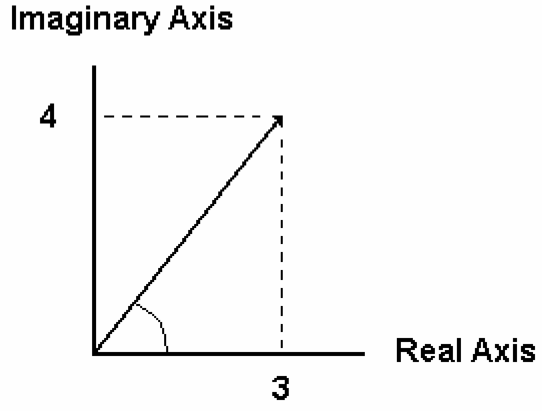

Figure 1: Complex Representation

As another illustration, consider an ideal inductor that has no real resistance, yet will have imaginary resistance (called reactance). An ideal capacitor acts in a similar manner, yet with a negative reactance. Over transmission lines the reactance increases due to the natural inductance of wire, thus the imaginary component of the voltage and current increases similar to the relationship displayed in Figure 1 (an inductor would actually create a negative imaginary component.) Electrical companies place large banks of capacitors in their systems to offset the inductive reactance caused by transmission lines, increasing the real component of electricity delivered to customers.

2.2. Phasor Notation

In sinusoidal AC systems, it is often tedious to calculate circuit voltages and currents using the standard tools of KVL, KCL and Ohm's Law because the signals are described as sinusoidal functions of time that repeat over time intervals.

It is easier computationally to convert the sinusoids into Phasor form prior to applying circuit analysis techniques. This form resembles complex electrical notation as both are described with a magnitude and an angle. However, the difference is that the angle in complex notation describes the relationship between real and imaginary components of a single component's magnitude while the angle in phasor notation describes the relationship between a component's magnitude and that of a reference AC signal. In small circuit this reference AC signal is typically the source.

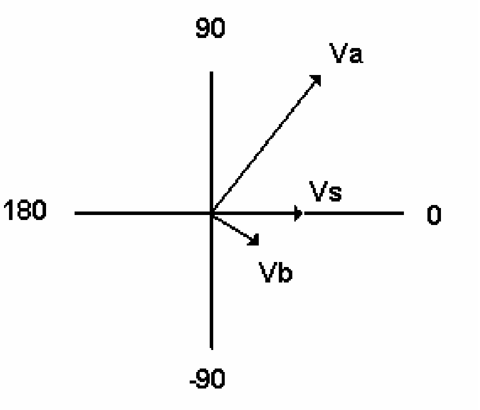

Figure 2: Phasor Representation

If we look to Figure 2 as an example, the source could have a magnitude of 2 Volts with a zero degree phase angle. The voltage across component A could have a magnitude of 4 Volts with a phase angle of 50 degrees with respect to the source. Similarly, the voltage across component B could have a magnitude voltage of 1 Volt with a phase angle of -30 degrees.

To convert an AC sinusoid into its corresponding phasor, it is necessary only to know its magnitude and phase angle. The magnitude of a pure sinusoidal AC signal will be its peak value (with respect to ground) divided by the square root of 2. This is called its Root-Mean-Square value (see previous lab for discussion of RMS).

Time Domain Example: $v(t) = 141.4\cos(377t + \pi/6)$ Volts

Voltage Magnitude: $\frac{141.4}{\sqrt{2}} = 100$ V_{RMS}

Voltage Phase Angle: $\frac{\pi}{6} * \frac{360}{2\pi} = 30°$

Voltage Phasor $V = 100\angle 30°$ V_{RMS}

If we take this voltage as the reference it could be written as $V = 100\angle 0°$ and a separate voltage of $v(t) = 141.4\cos(377t + \pi/4)$ would be written as $V = 100\angle 15°$ $\left(\frac{\pi}{4} * \frac{360}{2\pi} = 45° \right)$

Multiplying and dividing phasors is relatively simple. The phasor magnitudes will be modified as in any multiplication or division. However, when multiplying, the phase angles must be added and when dividing the phase angles must be subtracted.

Mathematical Phasor Example:

If $\vec{Z}_A = \vec{Z}_B * \frac{\vec{Z}_C}{\vec{Z}_D}$ then $\left|\vec{Z}_A\right| = \left|\vec{Z}_B\right| * \frac{\left|\vec{Z}_C\right|}{\left|\vec{Z}_D\right|}$ and $\angle \vec{Z}_A = \angle \vec{Z}_B + \angle \vec{Z}_C - \angle \vec{Z}_D$

2.3. Resonance

The resonant frequency, f_0, for an RLC series circuit is $f_0 = \frac{1}{2\pi\sqrt{LC}}$. At f_0, $\omega_0 L = \frac{1}{\omega_0 C}$. Thus, the circuit's total reactance, $j\omega L - \frac{j}{\omega C}$, is zero. Hence, the current, i, is in phase with the circuit's source voltage, V_s, the circuit's impedance is at a minimum, and the circuit's admittance (inverse of the impedance) is at a maximum.

2.4. PSpice® AC Analysis

An AC analysis performed by PSpice® or other simulation software will measure and display the magnitude of a current or voltage through or across a circuit element for a range of frequencies. In PSpice® schematics, create your circuit as normal but replace the sinusoidal sources with either IAC or VAC. Before performing the analysis go to Analysis → Setup. Enable AC sweep and modify the sweep properties to those desired or specified.

3. Prelab

The circuit in Figure 3 is a simple circuit consisting of a source, an inductor (with inherent resistance), a capacitor, and a resistor. Using theory, the complex resistance as seen by the source will be computed in both rectangular and polar form.

Given V_s is a Phasor of 5 Volts RMS and 0 degrees of Phase Angle (chosen as the reference phasor), calculate the resonant frequency, f_0, and find the impedance of the circuit as seen by the source for each of the frequencies in Table 1. Include your calculations used to find both rectangular and polar representations of Z_S for all three frequencies in your lab write-up. **Fill in Table 1 with your answers and obtain your lab instructor's signature before proceeding to the experimental procedure**.

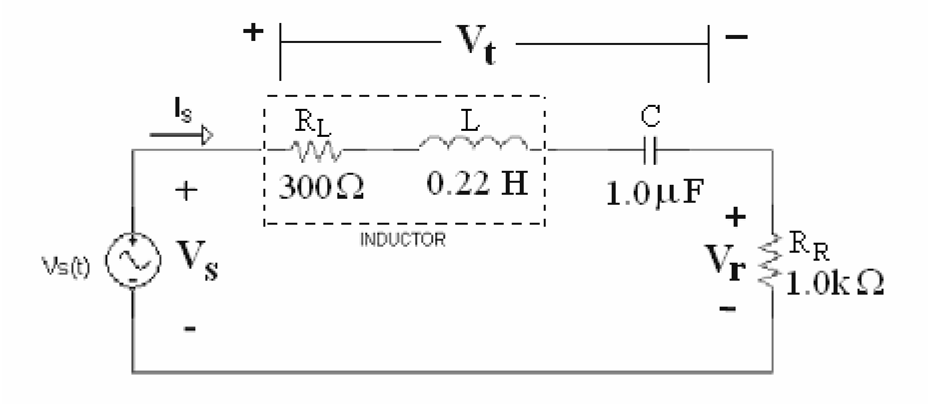

Figure 3: RLC Circuit

A couple hints:

- Impedances combine in the same manner as pure resistances.
- Reference "Complex Electrical Notation" in the background section of the lab for assistance in converting your rectangular representation of the impedance seen by the source to a polar form and "Resonance" for help with determining when there will be no phase shift between the voltage and current.

4. Experimental Procedure

4.1. Equipment

- Tektronix TDS 3012B Digital Phosphor Oscilloscope
- Agilent 33120A Waveform Generator
- Fluke 8050A Digital MultiMeter (DMM)
- Resistors, Inductors, Capacitors as Needed

4.2. RLC Measurements

Set the Digital Multimeter (DMM) to read AC Voltage and connect it across the output of the Function Generator. Select a Sinusoidal output on the Function Generator and adjust its output until the DMM reads 5 Volts RMS.

Connect the RLC circuit in Figure 3 using the obtained function generator settings.

Display V_s and V_r on the Oscilloscope with Channel 1 and 2, respectively. Set the Scope to trigger on a positive slope for Channel 1. Make sure that both channels have zero volts offset (both signals will be centered around the middle of the screen)

Adjust the frequency of the Function Generator until V_s and V_r are in phase (exactly the same zero crossings) and record this frequency as f_0 in Table 2.

Using the Math function of the Oscilloscope, measure V_t, where $V_t = V_s - V_r$ (that is, Ch1 – Ch2.)

Set the oscilloscope to automatically measure the voltage magnitude of the resistor, V_r, in RMS and the phase angle of V_r with respect to V_s for each frequency in Table 2. **Record these values**.

Set the oscilloscope to automatically measure the voltage magnitude of the inductor and capacitor, V_t, in RMS and the phase angle of V_t with respect to V_s for each frequency in Table 2. **Record these values** and take a Screenshot at each frequency to include in your lab write-up. *Make sure that the phase angle has the appropriate sign that agrees with the sign of your prelab results*.

Use the experimental data to determine rectangular and polar representations of the circuit impedance as seen by the source. Record your results in Table 3 and include calculations used to find both representations of Z_S for all three frequencies in your lab write-up.

4.3. Hints

- You know the vector $\vec{V}_S$ and the vector $\vec{V}_R$ (The vector is just the combination of the magnitude and phase angle recorded in Table 2). Using voltage division it can be determined that:

$$\vec{V}_R = \frac{\vec{Z}_R}{R_L + j(X_L + X_C)} * \vec{V}_S$$

- We will assume that the resistor is exactly R_R = 1000 ohms. Knowing that the resistor has no imaginary components it is known that $\vec{Z}_R = 1000\angle 0°$
- Use mathematical properties of vectors to solve for both polar and rectangular representations of the circuit impedance as seen by the source.

5. Simulated Procedure

Using PSpice® Schematics, perform an AC sweep analysis of the circuit displayed in Figure 3. Make sure to change the V_{MAG} setting for your voltage source to 5 Volts. Include a Screenshot of both the circuit constructed and a **logarithmic** plot of the resistor voltage from 0.5 f_0 to 2 f_0 in your lab write-up.

6. Conclusion

This concludes the lab. Make sure to return all components to their appropriate bins. The write-up should include all deliverables and explanations asked for in the procedure as well as the Data Entry and Lab Instructor Signature Page with all recorded numbers, equations, etc.

In addition, you should compare the theoretical calculations, experimental results and simulated measurements for the circuit impedance as seen by the source and comment on your observations (about these measurements and anything else you learned) from the lab.

Data Entry and Lab Instructor Signature Page(s)

Attach this/these page(s) to your write-up.

Table 1: Circuit Impedance as Viewed by Source (Prelab)

Frequency	**$Z_s = R_s + jX_s$**	**$\|Z_s\| \angle Z_s$**
$0.5\ f_0$		
f_0		
$2\ f_0$		

Prelab Signature ___________________

Table 2: Experimental Data for RLC Circuit in Figure 3

Frequency		**V_s**	**$\angle V_s$**	**V_r**	**$\angle V_r$**	**V_t**	**$\angle V_t$**
	Hz	*Volts*	*Deg.*	*Volts*	*Deg.*	*Volts*	*Deg.*
$0.5f_0$		5.0	0				
f_o		5.0	0				
$2f_o$		5.0	0				

Table 3: Calculated Date for RLC Circuit in Figure 3

Frequency	**$Z_s = R_s + jX_s$**	**$\|Z_s\| \angle Z_s$**
$0.5\ f_0$		
f_0		
$2\ f_0$		

Experimental Signature ___________________