AC Circuits

1. Objective

In this lab, the student will study sinusoidal voltages and currents in order to understand frequency, period, effective value, instantaneous power and average power. Also, the effective value of alternating waveforms will be measured and Ohm’s Law for AC circuits will be verified through the use of the Digital MultiMeter. Furthermore, students will become familiar with the use of a Wattmeter and with real and apparent power in AC circuits. Lastly, the student will explore the concept of phase angle and study the relationship between real and apparent power.

2. Background

2.1. Alternating Current (AC) Basics

Alternating Current (AC) is the world standard for driving motors and other electrical equipment. As the name implies, an alternating current continually and periodically changes direction, going first one way and then reversing. One may consider an alternating current to be a DC current, which is constantly and periodically changing amplitude and direction (conversely, a DC current is an AC current with a frequency of zero). The time necessary for the current to undergo one complete change of amplitude and direction is called a cycle. The number of cycles which occur in one second is called the frequency, which is measured in Hertz (cycles per second). In North America, AC system frequency is standardized at 60 Hertz. Much of the remaining world has chosen a 50 Hz standard.

Alternating voltages also reverse polarity in a periodic manner with continually changing amplitude. The shape of the voltage waveform is dependent upon the manner in which it is produced. One may construct a device, which produces voltage waveforms that are square waves, triangular waves, etc. However, one type of waveform is most suitable to the use of electric power in transformers and electric motors—the sine wave. Pure sinusoidal waveforms of a single frequency minimize mechanical and electrical losses in transformers and motors, thus allowing for the highest efficiency of energy conversion. If one considers the Fourier Transform of a triangular wave or square wave, which consist of a large number of sinusoidal waveforms at many frequencies, it becomes clear that there is increased potential for detrimental effects such as mechanical resonance, eddy currents, sequence currents, etc. A pure sine wave undergoing Fourier Transformation is left unchanged, minimizing losses. Another advantage of sine wave voltages is that the resulting current waveforms are also sinusoidal. This is not necessarily true of other waveform shapes. Having consistent wave shapes reduces the burden of power calculations and the analysis of electric systems.

2.2. Effective Value

See Lab 4 for a discussion of the effective/RMS value of an AC signal.
2.3. Wattmeters

In DC circuits the power supplied to a resistive load is always equal to the product of the voltage and current. In AC circuits that is not necessarily the case, even when using RMS (or effective) values of voltage and current. Impedance elements in AC circuits are not limited to resistances, but include reactive elements such as the capacitor and inductor. Reactive elements do not consume real power as a resistor does. Instead they absorb or produce reactive power and in the process change the voltage and current waveforms in the circuit. For this reason, wattmeters are an essential tool in the analysis of AC circuits.

Apparent (also known as complex) Power (measured in volt-amperes, VA) is defined as the product of the RMS AC voltage and RMS AC current. Apparent power (VA) is only equal to real power (Watts) when the load circuit is purely resistive, as in the previous laboratory experiments. Load circuits that are not purely resistive will have an apparent power significantly different from real power. A refrigerator is an example of a load which absorbs a large quantity of reactive power. Figure 1 depicts the relationship between real power, $P$, apparent power, $S$, and reactive power, $Q$. Since $S = I_{RMS} V_{RMS}$, it can be seen from the diagram that $P = S \cos(\theta) = VI \cos(\theta)$.

2.4. Phase Angle

In a DC circuit with a resistive load, an increase in DC voltage level produces a proportional increase in the DC current flowing in the circuit. This is also true in an AC circuit with a purely resistive load. An AC voltage applied to a resistor will produce an AC current whose instantaneous value corresponds directly to the instantaneous value of the voltage. The peak value of the current waveform will occur at exactly the same moment in time as the peak value of the voltage waveform. Likewise, the current zero crossing will occur at the same time as the voltage zero crossing. Such waveforms are said to be in phase. Voltage and current waveforms that are in phase are shown in Figure 2.
Most AC circuits, however, contain other elements in addition to pure resistances. Under such conditions, the current waveform will **not** be in phase with the voltage. In an AC circuit where the peak value of the current waveform occurs later in time than the peak of the voltage waveform, the current is said to **lag** the voltage. Such a condition is shown below in Figure 3.

Figure 4 depicts a current whose peak value occurs earlier in time than the peak value of the voltage. This current is said to **lead** the voltage. It is possible that the current could lead or lag the voltage by as much as 90 electrical degrees (\(\pi/2\) radians). In Figure 5, the current lags the voltage by 90 degrees. It is interesting to note that the current maximum occurs when the voltage is ZERO. Likewise, the voltage maximum occurs at the current zero. Although this seems highly
improbable, it occurs because the AC circuit consists of an element which is capable of storing and releasing energy, such as an inductor or capacitor. These elements absorb energy during part of the AC cycle and return it to the circuit later in the cycle. The storing and releasing of energy accounts for the phase angle difference between voltage and current. Loads that are purely inductive will have a current that lags voltage by 90 degrees. Loads that are purely capacitive will have a current that leads voltage by 90 degrees, such as Figure 6. In either case, a capacitive or inductive load will absorb power for one quarter cycle and then release it all back to the circuit the next quarter cycle. Capacitors and inductors are called reactive elements and as such, dissipate little real power.

3. Prelab

Consider the ideal AC generator shown in Figure 7. Assume it produces a sinusoidal voltage output waveform between terminals A and B. Let the voltage waveform have a peak value of 70 Volts, such that the algebraic description is $v(t) = 70 \sin (\omega t)$ volts. Calculate the value of $v(t)$ at 15 degree intervals (i.e. when $\omega t = 0^\circ$, $15^\circ$, $30^\circ$, ..., $330^\circ$, $360^\circ$) and plot them in Graph 1 provided. Also, record your values in Table 1, since you will need these values again. Connect the plotted points with a smooth curve and label it $v(t)$. Remember that the waveform will have a negative value for half its period.

If a load resistance of 2 Ohms is connected across terminals A and B, a current, $i(t)$, will flow. Knowing the instantaneous value of the voltage from the graph and using Ohm’s Law, calculate and record the instantaneous values of $i(t)$ for the values of $\omega t$ in Table 2.
Plot the instantaneous current values recorded in Table 2 on the same graph with \( v(t) \) (Graph 1) and draw a smooth curve through the plotted points. Label this curve \( i(t) \).

Knowing that the instantaneous power, \( p(t) \), is the product of the instantaneous voltage and current, calculate \( p(t) \) at every 30 degree interval and record your values in Table 3.

Plot the instantaneous power values on the same graph with \( v(t) \) and \( i(t) \) (Graph 1), then draw a smooth curve through the plotted points (note: the scale on the right side of the graph corresponds to the power values). Label this curve \( p(t) \).

Examine the power curve and determine the maximum, or peak, instantaneous power dissipated by the resistor and the minimum value of instantaneous power. Also make your best estimate as to the average power dissipated by the resistor. Record these values in Table 4.

Answer questions P.1 — P.6 and record your answers in Table 5.

P.1: It can be shown mathematically that the average power will equal exactly one-half the peak instantaneous power. The average power is equivalent to that supplied to the two ohm resistor from a DC source. If 2500 Watts were dissipated by a 2 Ohm resistor, supplied by a DC source, what would the DC Voltage of the supply be?

P.2: What is the DC current supplied by the source for those conditions?

Note that the AC values of 100 Volts, peak and 50 Amps, peak deliver the same average power as DC values of 70.7 Volts, DC and 35.4 Amps, DC. This is a ratio of one over the square root of two \((1/\sqrt{2})\). Thus, the effective value of an AC signal is its peak value divided by the square root of two. This is known mathematically as the Root-Mean-Square or RMS value of the waveform, and leads to the following equations:

\[
V_{\text{RMS}} = 0.707 \ V_{\text{Peak}} \quad I_{\text{RMS}} = 0.707 \ I_{\text{Peak}}
\]

P.3: How long does it take, in seconds, for the voltage to change from 0 to maximum (peak) value on a 60 Hz power system?
P.4: What is the length of time of the positive portion of one complete cycle of a 60 Hz current waveform (assuming there is no DC offset)?

P.5: In a 50 Hz system, what is the time duration of one complete cycle?

P.6: If standard residential wiring is rated at 120 Volts, RMS, what is its peak value?

4. Experimental Procedure

4.1. Equipment

- Agilent 33120A Waveform Generator
- Fluke 8050A Digital MultiMeter (DMM)
- Resistors, Capacitors, Inductors As Needed

4.2. AC Voltage and Current Measurements

Select a 56 Ω resistor, a 100 Ω resistor, a 220 mH inductor, and a 10 uF capacitor from the component bins. Measure their resistances with the Digital MultiMeter (DMM) (including the capacitor and inductor). If desired, you can also measure the inductance and capacitance of the elements. Connect the output of the function generator to the DMM, set the function to a 60 Hz sine wave, and adjust the amplitude until the DMM reads 3 $V_{RMS}$.

Connect the circuit shown in Figure 8 where the “element” is first the 56 Ω resistor. Include the DMM in the circuit and measure the AC current through the resistor 56 Ω resistor. Repeat this process by replacing the 56 Ω resistor with the 100 Ω resistor, then the inductor, and then the capacitor. Lastly connect the circuit so that a series combination of the 100 Ω resistor and the capacitor fills in the “Elements” box of Figure 8 and repeat the process. Switch the amplitude of the sine wave such that the DMM measures 5 $V_{RMS}$ when directly connected to the generator and repeat the procedure for of the previous five scenarios and record all your results in Table 6.

![Figure 8: Simple AC Circuit](image)
Calculate the resistance for each of the two resistors using $V_{RMS}$ and $I_{RMS}$ and Ohm’s law. **Record the results in Table 6** in the row marked $V / I$. Also, calculate the impedance for the inductor and capacitor $(V / I)$.

Calculate the complex/apparent power, $S$, in each configuration and **record the results in Table 6** in the row marked $V \times I$. Lastly, calculate the real power for each case and record this in the appropriate two rows. In order to do this, it might be necessary to compute the phasors for the voltage, current, and impedance (where the voltage can be used as a reference and therefore will have an angle of zero) using the equations from lab 10 ($V = IZ$, $Z_L = j\omega L$, etc.).

**Note:** When answering the questions in Section 7 corresponding to this section, remember that the internal resistance of the function generator (50 $\Omega$) must be included in the total resistance since it affects the current. Also, the equation $P = VI$ is the equation for power in DC circuits. In AC circuits, it is the equation for $S$, the apparent power. Real power is calculated using $P = VI\cos\theta$ and is equivalent to the reading that would be measured on a Wattmeter. However, in the case where the load circuit is purely resistive, such as the first two scenarios above, the apparent power, $S$, is equal to the real power, $P$ (for the pure resistance case, $\theta = 0$ and hence $\cos\theta = 1$).

**5. Theoretical Measurements**

The ratio of real power, $P$, to apparent power, $S$, is called the power factor, $pf$, for the AC circuit and is expressed as the equation, $pf = P/S$. Calculate the power factor for each of the five circuits analyzed above and **record your results in Table 7**.

The load for the circuit shown below is purely resistive. The AC Ammeter reads 35.3 $A_{RMS}$ and the AC Voltmeter reads 70.7 $V_{RMS}$. Calculate the apparent power, $S$, supplied to the load. ($S = VI$)

![AC Circuit Diagram](image)

$V_s = 70.7$ $V$ (RMS)

$I_s = 35.3$ $A$ (RMS)

$S = V_sI_s = \text{_______} \text{ VA}$

**Figure 9: Wattmeter Measurement**

The voltage and current waveforms for this circuit are shown in Figure 10. The instantaneous power, $p(t)$, is also plotted in Figure 10. Notice that the power waveform is sinusoidal but at double the frequency (two cycles of power per one cycle of voltage or current). Recall the trig identity for $\sin^2A$ and when you answer question 15 in Section 7.
The load for the circuit in Figure 11 is purely capacitive. The current for this load leads the voltage by 90 degrees, such as in Figure 6. The AC Ammeter still reads 35.3 Amps RMS and the AC Voltmeter still reads 70.7 V RMS. Calculate the apparent power, $S$, supplied to the load.

$$V_s = 70.7 \text{ V rms}$$
$$I_s = 35.3 \text{ A rms}$$
$$S = V_s I_s = \phantom{0}$$ VA

The voltage and current waveforms for this circuit are shown in Graph 2. Notice that the current maximum occurs at a voltage zero and vice-versa.

Table 8 contains $v(t)$ and $i(t)$ values at 45 degree intervals. Complete the table by calculating the instantaneous power, $p(t) = v(t)i(t)$ VA at each interval.

Plot the calculated values of $p(t)$ on Graph 2 and draw a smooth curve connecting the points. Remember that the power curve will be sinusoidal at twice the frequency of the voltage and current.

Determine the information from the power curve in questions 18 through 20.
The load for the circuit shown in Figure 12 is purely inductive. The current for this load lags the voltage by 90 degrees, such as in Figure 5. The AC Ammeter still reads 35.3 Amps RMS and the AC Voltmeter still reads 70.7 V RMS. Calculate the apparent power, S, supplied to the load.

\[
\begin{align*}
V_s &= 70.7 \text{ V rms} \\
I_s &= 35.3 \text{ A rms} \\
S &= V_s I_s = _______ \text{ VA}
\end{align*}
\]

![Figure 12: Wattmeter Measurements for Inductive Load](image)

The voltage and current waveforms for this circuit are shown in Graph 3. Notice that the current maximum occurs at a voltage zero and vice-versa.

Table 9 contains \(v(t)\) and \(i(t)\) values at 45 degree intervals. **Complete the table** by calculating the instantaneous power, \(p(t) = v(t)i(t)\) VA at each interval. **Plot the calculated values of** \(p(t)\) **on Graph 3** and draw a smooth curve connecting the points. Remember that the power curve will be sinusoidal at twice the frequency of the voltage and current. Complete questions 21 through 23 in Section 7 for Graph 3.

**6. Conclusion**

This concludes the lab. Make sure to return all components to their appropriate bins. The write-up for this lab will be a memo and should include all deliverables and explanations asked for in the procedure as well as the Data Entry and Lab Instructor Signature Page with all recorded numbers, equations, etc. When answering the questions in Section 7, you may simply write on the paper and include them in an appendix if you wish and refer to where they are in the body of the memo or you can also type them.
7. Questions

1) T/F If in one complete cycle, the instantaneous power, p(t), is always positive, then the load must only be composed of resistors. Why? Why not?

2) Make a rough sketch of a current waveform lagging a voltage waveform by 60 degrees.

3) If there was no output impedance from the function generator in the experiment, would there be any significant real power dissipation in the capacitor? In the inductor? Explain.

4) Do the results of Subsection 4.2 support or disprove Ohm’s law with respect to resistors in AC circuits? Why are there differences in the V / I ratio for the inductor and capacitor?

5) Explain the difference between apparent power, S, and real power, P. In what units does one express apparent power? Real power?
6) A Wattmeter reads zero watts when the current lags or leads the voltage by 90 degrees. Explain this phenomenon.

7) In a 60 Hz system, if the current leads the voltage by 45 degrees, what is the time difference, in milliseconds, between the occurrence of each respective peak?

8) An incandescent lamp rated at 100 Watts, emits a certain amount of illumination when energized at 120 V_{RMS}. Would the amount of illumination increase, decrease or remain the same when the lamp is energized at 120 V_{DC}? Explain.

9) Explain in your own words what is meant by the terms effective voltage and effective current.

10) What might be the explanation for zero readings on the DC Ammeter and Voltmeter when measuring AC signals?

11) If an AC distribution system operates at 600 Volts, AC, what is the peak value of the voltage?
12) A 60 Watt incandescent lamp is operated at 120 V_{RMS}. Calculate the AC current and the 'hot' resistance of the lamp (i.e. when it is on and consuming 60 Watts).

13) Try to name two household devices that should have a high power factor and one device which should have a low power factor.

14) What should the wattmeter read for the circuit in Figure 9?

15) Does the power curve ever have a negative value for a resistive load? Is the power real? Write the trigonometric identity which proves that the average power is actually $\frac{1}{2}$ the peak power.

16) What is the apparent power for the circuit in Figure 11? Should the wattmeter indicate the same power?

17) What is the apparent power for the circuit in Figure 12? Should the wattmeter indicate the same power?

18) For Graph 2, what is the peak power? What phase angle does this correspond to?

19) For Graph 2, is the instantaneous power ever negative? Are all the peaks of the power curve of the same magnitude?

20) For Graph 2, does the area enclosed under the positive portions of the curve equal the area enclosed under the negative portions of the curve? What is the average or real power for one complete cycle (360 degrees)?
21) For Graph 3, what is the peak power? What phase angle does this correspond to?

22) For Graph 3, is the instantaneous power ever negative? Are all the peaks of the power curve of the same magnitude?

23) For Graph 3, does the area enclosed under the positive portions of the curve equal the area enclosed under the negative portions of the curve? What is the average or real power for one complete cycle (360 degrees)?
Graph 1: Sinusoidal Plot [Prelab]

Table 1: Sinusoid Voltage Values (in Volts) [Prelab]

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Table 2: Sinusoid Current Values (in Amps) [Prelab]

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Table 3: Sinusoid Power Values (in Watts) [Prelab]

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Table 4: Important Power Values (in Watts) [Prelab]

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<th>Maximum Power</th>
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<table>
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<tr>
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<th>#3 (Sec)</th>
<th>#4 (Sec)</th>
<th>#5 (Sec)</th>
<th>#6 (Volts)</th>
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Table 5: Answers to Questions P.1 – P.6 [Prelab]

Table 6: Ohm’s Law for AC Circuits

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<td>100 $\Omega$</td>
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<tr>
<td>$V / I (\Omega)$</td>
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<td></td>
</tr>
<tr>
<td>$S = V \times I (VA)$</td>
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<tr>
<td>$P = V I \cos(\theta)$</td>
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<table>
<thead>
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<td>$V / I (\Omega)$</td>
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<td>$P = V I \cos(\theta)$</td>
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Table 7: Power Factor Calculations

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<th>Power Factor, $pf = \frac{\text{Real Power, } P}{\text{Apparent Power, } S}$</th>
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<tr>
<td>$R$ (56 $\Omega$)</td>
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<td>$pf = \frac{P}{S}$</td>
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Table 8: Plotting Points for Capacitive Load

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<th>Degree</th>
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Graph 2: Current Leads Voltage by 90° (Capacitive)

Graph 3: Current lags Voltage by 90° (Inductive)

Table 9: Plotting Points for Inductive Load

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Lab Signature __________________________

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ECE Department 16 April 23/25, 2007