

Equivalent Equipment Circuits

1. Introduction

The student will analyze the internal properties of the equipment used in lab. The input resistance of the oscilloscope and Digital MultiMeter (DMM) when used as a voltmeter will be measured. The output resistance of the function generator will similarly be determined. The student will also determine the Thevenin and Norton equivalents of a complex circuit using PSpice®.

2. Background

When an electrical instrument is connected to a circuit to provide power or take measurements it becomes part of the circuit. Often the resistance of the connected instruments is neglected as they have been designed to not interfere with most circuits. Even though electricity flows through multiple elements inside of the instrument, these components may be modeled as a simple resistor or resistor and source.

To determine the internal resistance of an instrument it is usually only necessary to vary a single component of an exterior connected circuit. Enough measurements are available throughout the exterior circuit to provide information for basic circuit analysis techniques to calculate the internal properties of an instrument.

2.1. Root Mean Square (RMS)

The current and voltage in alternating current (AC) systems is not constant. Thus, one cannot easily apply ohm's law to a circuit with an AC source. If one thinks of a resistive element, current traveling forward will heat the element up just as much as current traveling backwards. If one desires to use Ohm's law to analyze a circuit with an AC source, RMS values for voltage and current must be calculated or measured.

An equation for this value can be derived by examining the average power. For resistive circuits, the instantaneous power can be expressed as:

$$P_{inst}(t) = v(t)i(t) = \frac{v^2(t)}{R}$$

When the voltage is periodic (e.g. a sinusoidal wave), the average power can then be expressed as:

$$P_{avg} = \text{avg} \left(\frac{v^2(t)}{R} \right) \text{over 1 period} = \frac{1}{T} \int_{t_0}^{t_0+T} \frac{v^2(t)}{R} dt$$

If the input waveform is periodic, the average power will be constant. As a result, there is a DC voltage which would produce the same average power as a given periodic waveform.

This voltage is termed the Root-Mean-Square voltage— V_{RMS} . Since V_{RMS} is a constant voltage, the calculation of the average power is simply,

$$P_{avg} = \left(\frac{V_{RMS}^2}{R} \right)$$

Setting the two average power equations equal to each other and solving for V_{RMS} produces:

$$V_{RMS} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} v^2(t) dt}$$

which is the square root of the mean of the squared voltage (hence the name).

For single sinusoidal voltages, this can be shown to be the amplitude (A) of the sinusoid divided by the square root of two, i.e.

$$V_{RMS} = \frac{A}{\sqrt{2}}$$

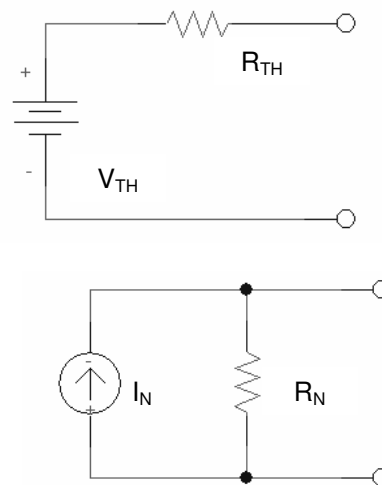
This can be shown graphically by noting the average of the square of a sinusoid is simply $0.5A^2$ (since the maximum value of the waveform is A^2 , the minimum is zero, and the waveform spends equal amounts of time above and below the average value). Taking the square root of this then gives us the previous equation.

2.2. Thevenin and Norton Equivalents

Thevenin's theorem states that a two terminal circuit can be replaced by an equivalent circuit consisting of a voltage source, V_{TH} , in series with a resistor, R_{TH} , where V_{TH} is the open-circuit voltage, V_{OC} , at the terminals and R_{TH} is equivalent to the resistance at the terminals when all independent sources are turned off (see the equations below for the calculation of the resistance when the circuit contains both independent *and dependent* sources).

Norton's theorem states that a two terminal circuit can be replaced by an equivalent circuit consisting of a current source, I_N , in parallel with a resistor, R_N , where I_N is the short-circuit current, I_{SC} , through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off. Mathematically these relationships can be described as in following equations:

$$V_{TH} = V_{OC} \quad I_N = I_{SC} \quad R_{IN} = R_{TH} = R_N = (V_{TH} / I_N)$$



3. Procedure

3.1. Equipment

- Tektronix TDS 3012B Digital Phosphor Oscilloscope
- Agilent E3631A DC Power Supply
- Agilent 33120A Waveform Generator
- Fluke 8050A Digital MultiMeter (DMM)
- Resistors as Needed

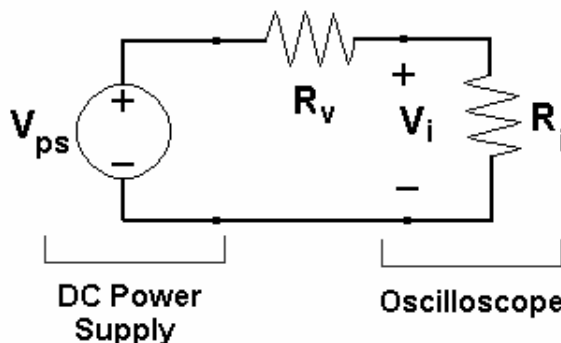


Figure 1: Input Resistance Measurement

3.2. Internal Resistance of the Oscilloscope

Connect the oscilloscope directly to the DC power supply. In this manner the circuit in Figure 1 is constructed with the variable resistance R_v set to 0 volts and R_i denoting the internal resistance of the oscilloscope.

Adjust the DC power supply until the oscilloscope measures 8 volts. **Record the corresponding value of the DC power supply in Table 1.**

Select a nominal 10 M Ω resistor and measure its resistance using the Digital MultiMeter (DMM). **Record this value in Tables 1 and 2.**

Using the 10 M Ω resistor as R_v , construct the circuit displayed in Figure 1 and measure the voltage across the oscilloscope terminals V_i (with the same value for V_{ps}). **Record this value in Table 1.**

Knowing three of the four variables of the circuit displayed in Figure 1, calculate the internal resistance of the oscilloscope (denoted as R_i in Figure 1) and **record this value in Table 1.** Include this calculation in lab write up.

3.3. Internal Resistance of the MultiMeter Used as a Voltmeter

Set the DMM to measure voltage and connect it directly to the DC power supply. In this manner the circuit in Figure 1 is constructed with the variable resistance R_v set to 0 volts.

Adjust the DC power supply until the DMM reads 16 volts. Make sure the DMM is set to measure DC, not AC voltages. **Record the corresponding value of the DC power supply in Table 2.**

Using the (nominally) 10 M Ω resistor as R_v , construct the circuit displayed in Figure 1 and measure the voltage across the DMM terminals V_i . **Record this value in Table 2.** Knowing three of the four variables of the circuit displayed in Figure 1, calculate the internal resistance of the DMM (denoted as R_i in Figure 1) and **record this value in Table 2.** Include this calculation in your lab write-up.

3.4. Output Resistance of the Function Generator

Connect the DMM directly to the function generator. In this manner the circuit in Figure 2 is constructed with the load resistance R_L set to 0 volts. We will neglect the internal resistance of the voltmeter for the purposes of this experiment and assume that the circuit is open, thus V_O is equal to V_{fg} with R_L removed.

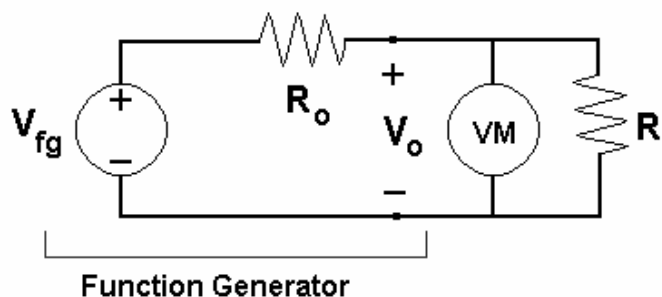


Figure 2: Output Resistance Measurement

Make sure the DMM is set to measure AC RMS, not DC. Adjust the function generator to a sinusoidal frequency of 50Hz. Adjust the amplitude until the DMM displays 1.6 Volts. **Record the corresponding value of the function generator amplitude in Table 3.**

Select a nominal resistor of about 50 Ω , measure its resistance using the DMM, and **record this value in Table 3.**

Using the 50 Ω resistor as R_L , construct the circuit displayed in Figure 2 and measure the voltage displayed on the DMM (V_O). **Record this value in Table 3.**

Knowing three of the four variables of the circuit displayed in Figure 2, calculate the output resistance of the function generator (denoted as R_O in Figure 2) and **record this value in Table 3.** Include this calculation in your lab write-up.

3.5. Norton and Thevenin Analysis by means of SPICE

Assume that the circuit represented in Figure 3 can only be analyzed by taking measurements at the open terminals represented by the voltage V_a . Create a PSpice[®] schematic to measure the open circuit voltage, V_{OC} , at the open terminals of the circuit displayed in Figure 3. **Record this value in Table 4.**

Copy and paste this circuit and then modify the schematic to measure the short circuit current, I_{SC} , between the open terminals. **Record this value in Table 4** and include a screenshot of the schematic with the two circuits in your write-up.

Knowing the open circuit voltage, V_{OC} , and the short circuit current, I_{SC} , **fill in the remainder of Table 4.**

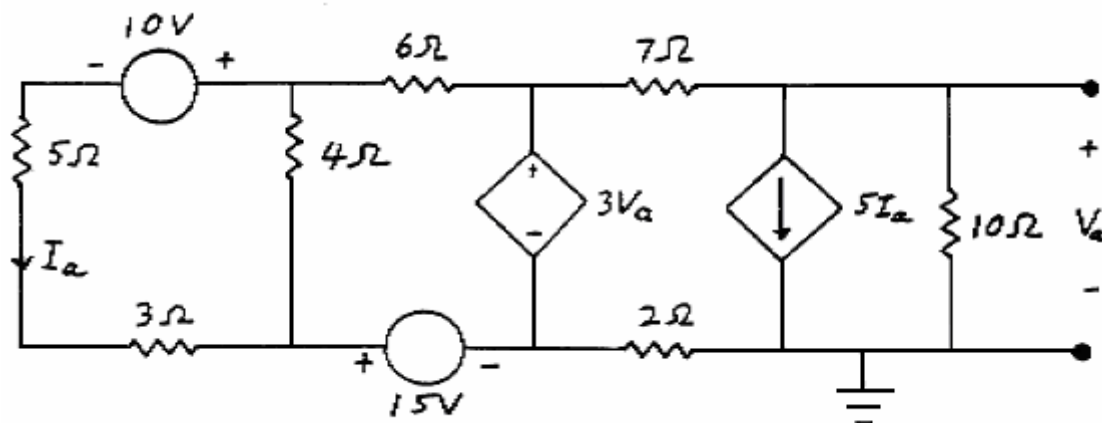


Figure 3: DC Circuit for SPICE Analysis

4. Conclusions

This concludes the lab. Make sure to return all components to their appropriate bins. This week you are not required to complete a full report. Instead, it is sufficient produce a memo which includes the following:

- Greeting, salutation, etc. as defined in Lab 2.
- The Data Entry and Lab Instructor Signature Page with all recorded numbers.
- Any snapshots, circuit diagrams, and/or calculations requested in the procedure.
- Answers to the questions in Subsection 4.1.

4.1. Questions

1. When measuring the internal resistance of the oscilloscope and DMM we did not take into effect the output resistance of the DC power supply. Did this make a substantial difference in your calculated results? Why or why not?
2. When measuring the internal resistance of the function generator we did not take into effect the internal resistance of the oscilloscope even though it was connected in parallel across R_L . Does this make a substantial difference in your calculated results? Describe why or why not mathematically utilizing values for R_L and the value determined to be the internal resistance of the oscilloscope.
3. It is common to find 10X oscilloscope probes that have ten times the impedance of a normal probe. In what circumstances would one use a 10X probe? Would there be any drawbacks to always using a 10X probe?
4. Norton's and Thevenin's theories were clearly applied in the PSpice[®] portion of the lab exercise. However, this theory was applied earlier in the lab. Describe where and how this theory was applied.

Data Entry and Lab Instructor Signature Page

Attach this page to your write-up.

Table 1: Values for the Oscilloscope Resistance Measurement

V_{PS} (V)	R_V (Ω)	V_i (V)	R_i (Ω)

Table 2: Values for the Internal Resistance of the Voltmeter

V_{PS} (V)	R_V (Ω)	V_i (V)	R_i (Ω)

Table 3: Measured Values for the Output Resistance of the Function Generator

V_{fg} (V_{RMS})	R_L (Ω)	V_O (V_{RMS})	R_O (Ω)

Signature for Tables 1 – 3 + Student explains RMS
and how Norton's and/or Thevenin's Theorem(s) were applied _____

Table 4: PSpice[®] Values

V_{OC} (V)	I_{SC} (A)	V_{Th} (V)	R_{Th} (Ω)	I_N (A)	R_N (Ω)