RC & RL Transient Response

1. Introduction

The student will analyze series RC and RL circuits. A step input will excite these respective circuits, producing a transient voltage response across various circuit elements. These responses will be analyzed by theory, simulation and experimental results. The primary response properties of concern are time constant, initial value and final value.

The equations that govern RC and RL circuit transient responses will be calculated by the student, both forward using theory and backwards after having observed experimental results. Methods to measure the time constant of an experimental system and produce a step input using a function generator will be shown.

2. Background

2.1. Equations for RC Circuits

- Time Constant: \( \tau_C = RC \)
- Capacitor Voltage Transient Equation: \( v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]e^{-t/\tau_C} \)

2.2. Equations for RL Circuits

- Time Constant: \( \tau_L = L/R \)
- Theoretical Inductor Instantaneous Voltage: \( v_L = L \frac{di_L}{dt} \)
- Inductor Current Transient Equation: \( i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-t/\tau_L} \)
- Inductor Voltage Transient Equation: \( v_L(t) = v_L(\infty) + [v_L(0) - v_L(\infty)]e^{-t/\tau_L} \)

2.3. Important Points to Consider

- The order of components is arbitrary in a series circuit.
- Capacitor voltage cannot change instantaneously.
- Inductor current cannot change instantaneously.
- An ideal unit step is zero volts until time zero whereas it instantaneously jumps to one volt and remains at one volt thereafter.
- A function generator has output impedance.
- A real inductor has both resistive and inductive components. Writing a voltage transient equation for a real inductor requires adding these two components together. Equations for \( v_L \) assume ideal inductor, thus the value of the inductor’s resistance must be multiplied by the inductors current and must be added to \( v_L \) to find the real inductor’s voltage transient equation.
- The voltage of an ideal inductor at \( t=0^+ \) will not be 0.
3. Prelab

3.1. Voltage Transient Response in RC Components Due to a Unit Step

Suppose a unit step occurs at time $t=0$ in the RC circuit displayed in Figure 1. Calculate the initial voltage across the capacitor, $v_C(t=0^+)$, final voltage across the capacitor, $v_C(t=\infty)$, initial voltage across the 680Ω resistor, $v_R(t=0^+)$, final voltage across the 680 Ω resistor, $v_R(t=\infty)$, and the time constant, $\tau$, of the circuit. Using nominal component values, calculate the voltage transient time response equation for the capacitor, $v_C(t)$, and voltage transient time response equation for the resistor, $v_R(t)$, both for $t > 0$. Fill in your values in Table 1.

![Figure 1: RC Circuit](image)

3.2 Voltage Transient Response in RL Elements Due to a Unit Step

Suppose a unit step occurs at time $t=0$ in the RL circuit displayed as Figure 2. Calculate the initial voltage across the inductor, $v_L(t=0^+)$, final voltage across the inductor, $v_L(t=\infty)$, initial voltage across the 680Ω resistor, $v_R(t=0^+)$, final voltage across the 680 Ω resistor, $v_R(t=\infty)$, and the time constant, $\tau_L$, of the circuit. Using nominal component values, calculate the voltage transient time response equation for the inductor, $v_L(t)$, and voltage transient time response equation for the resistor, $v_R(t)$, both for $t > 0$. Record your results in Table 2. Have your lab instructor verify these values and obtain his/her signature at the beginning of lab.

![Figure 2: RL Circuit](image)
4. Experimental Preparation

4.1. Imitating a unit step

We do not provide the equipment to produce and analyze the response of a single unit step. We model a unit step by generating a square wave with a period much greater than the time constant ($\tau$) of the circuit (as was done in an earlier lab). This provides enough time for the circuit to settle before another imitated unit step is initiated.

The square wave generated should be 0 volts for $10\tau$ and 1 Volt for another $10\tau$. Thus, the square wave period will be $20\tau$ with a corresponding frequency of $1/(20\tau)$. An amplitude of 1 Volt along with a DC offset of 0.5 Volts must be set to ensure proper effect.

4.2. Measuring the Time Constant

The time constant is defined as the ratio $1 - e^{-1}$ of the rise or fall to the final value. This corresponds to approximately 63% of the rise or fall to the final value. The voltage corresponding to one time constant is $v_r = [v(\infty) - v(0)] * [1 - e^{-1}] + v(0)$. The time constant can be computed by finding the time it takes to reach $v_r$. With one of the oscilloscope’s vertical bars at the beginning of the unit step and one at $v_r$, the time difference will be displayed as $\Delta T$. A similar method can be used with the PSpice® cursors.

5. Experimental Procedure

5.1. Equipment

- Tektronix TDS 3012B Digital Phosphor Oscilloscope
- Agilent 33120A Waveform Generator
- Resistors, Inductors, Capacitors as Needed

5.2. Voltage Transient Response in RC Components Due to a Unit Step

Construct the circuit in Figure 1. The function generator should model a unit step as described in Subsection 4.1. Measure the voltage across the capacitor with one channel probe and the voltage across the function generator with the other channel probe. Enable the MATH function to display the voltage across the 680Ω resistor.

Using the oscilloscope horizontal bars, measure the initial capacitor voltage, final capacitor voltage, initial 680Ω resistor voltage, final 680Ω resistor voltage and determine the time constant. Lastly, write the corresponding transient equations describing the voltages for $t > 0$ (for one unit step). **Record these values in Table 3.**
Include an Oscilloscope Screenshot displaying both the capacitor and resistor voltage transient responses to a unit step in your write up.

5.3. Voltage transient response in RL elements due to a unit step

Construct the circuit in Figure 2. The function generator should model a unit step as described in Subsection 4.2. As with the RC circuit, place one channel probe across the unit step and the other across either the inductor or the 680Ω resistor utilizing the MATH function in a similar manner. The circuit components may be arranged in a different manner than that shown in Figure 2 for convenience.

Measure the initial inductor voltage, final inductor voltage, initial 680Ω resistor voltage, final 680Ω resistor voltage, and determine the time constant. Lastly, write the corresponding transient equations describing the voltages for \( t > 0 \) (for one unit step). Record these values in Table 4.

Include an Oscilloscope Screenshot displaying both the inductor and resistor voltage transient responses to a unit step in your write-up.

6. Simulated Procedure

6.1. Voltage Transient Response in RC Components Due to a Unit Step

Generate a schematic modeling the RC circuit’s voltage transient response to a unit step. Using the PSpice® cursor, measure the initial capacitor voltage, final capacitor voltage, initial 680Ω resistor voltage, final 680Ω resistor voltage and determine the time constant. Lastly, write the corresponding transient equations describing the voltages for \( t > 0 \) (for one unit step). Record these values in Table 5.

Include a schematic screenshot and a screenshot of the transient analysis of the voltage across the function generator, the capacitor and the resistor. These three traces should be displayed on the same graph.

6.2. Voltage Transient Response in RL Elements Due to a Unit Step

Generate a computer simulation modeling the RL circuit’s voltage transient response to a unit step. Measure the initial inductor voltage, final inductor voltage, initial 680Ω resistor voltage, final 680Ω resistor voltage, and determine the time constant. Lastly, write the corresponding transient equations describing the voltages for \( t > 0 \) (for one unit step). Record these values in Table 6.

Include a schematic screenshot and a screenshot of the transient analysis of the voltage across the function generator, the inductor and the resistor. These three traces should be displayed on the same graph.
7. Conclusions

This concludes the lab. Make sure to return all components to their appropriate bins. This week you are not required to complete a full report. Instead, you will be writing a memo. The memo should include:

- The recipient (i.e. TA) and his/her general company (University) information
- The writer (you) and your general company (University) information
- The date
- The purpose of writing the memo (i.e. conveying your lab results)
- A brief discussion of your results in a clear manner
- Greeting and conclusion

The discussion within the memo should include an explanation for the reasons for discrepancies between the theoretical, simulated and experimental values for $v_C(t=0^+)$, $v_C(t=\infty)$, $v_L(t=0^+)$, $v_L(t=\infty)$, $v_R(t=0^+)$, $v_R(t=\infty)$ and $\tau$.

The memo should also include the following as attachment and refer to them in the body of the memo:

- The Data Entry and Lab Instructor Signature Page with all recorded numbers.
- Any snapshots, circuit diagrams, and/or calculations requested in the procedure.
Data Entry and Lab Instructor Signature Page(s)
Attach this/these page(s) to your write-up.

Table 1: Theoretical RC Circuit Values (Prelab)

<table>
<thead>
<tr>
<th>$v_c(t=0^+)$ (V)</th>
<th>$v_c(t=\infty)$ (V)</th>
<th>$v_R(t=0^+)$ (V)</th>
<th>$v_R(t=\infty)$ (V)</th>
<th>$\tau_c$ (s)</th>
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Transient Equation (Capacitor):

Transient Equation (Resistor):

Table 2: Theoretical RL Circuit Values (Prelab)

<table>
<thead>
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<th>$v_L(t=0^+)$ (V)</th>
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Transient Equation (Inductor):

Transient Equation (Resistor):

Prelab Completion Signature
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Table 3: Measured RC Circuit Values

<table>
<thead>
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<th>$v_c(t=0^+)$ (V)</th>
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Transient Equation (Capacitor):

Transient Equation (Resistor):

Table 4: Measured RL Circuit Values

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Transient Equation (Inductor):

Transient Equation (Resistor):
Table 5: Simulated RC Circuit Values

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<th>$\tau_c$ (s)</th>
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Transient Equation (Capacitor):

Transient Equation (Resistor):

Table 6: Simulated RL Circuit Values

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<th>$V_R(t=\infty)$ (V)</th>
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Transient Equation (Inductor):

Transient Equation (Resistor):