

ECE2006 LABORATORY 9

FREQUENCY RESPONSE OF RLC CIRCUITS

OBJECTIVES

The learning objectives for this laboratory are to give the student the ability to:

- Experimentally determine the resonant frequency
- Use the oscilloscope to display the difference of two AC signals
- Use the oscilloscope to measure phase angle differences between AC signals
- Use the digital multimeter to measure AC voltage magnitudes
- Calculate impedances from phase and magnitude measurements
- Use the PSPICE AC analysis tool to analyze AC circuits

REFERENCES

- Alexander/Sadiku, “Fundamentals of Electric Circuits – 2nd Edition”, 2003, McGraw-Hill
- See above reference, Appendix D.4: pp.A-38 thru A-46, for **PSPICE** analysis of RLC transient response.

BACKGROUND

Phasors

In sinusoidal AC systems, it is often tedious to calculate circuit voltages and currents using the standard tools of KVL, KCL and Ohm’s Law because the signals are described as sinusoidal functions of time.

It is easier computationally to convert the sinusoids into Phasor form prior to applying circuit analysis techniques. To convert an AC sinusoid into its corresponding phasor, it is necessary only to know its magnitude and phase angle.

The magnitude of a pure sinusoidal AC signal will be its peak value (with respect to ground) divided by the square-root of 2. This is called its Root-Mean-Square value.

Example: $v(t) = 141.4 \cos(377 t + \pi/6)$ Volts (a voltage in Time Domain)

Voltage Magnitude = $141.4/\sqrt{2} = 100$ Volts RMS

Voltage Phasor , $V = 100 /_{+30^\circ}$ Volts

The phase of a pure sinusoidal AC signal must be measured with respect to a reference, another AC signal. If no other AC signal is of interest, one can assume that the signal being studied is the reference and therefore its phase angle is zero. In the example above, reference information is given as $[+ \pi/6]$. Therefore, the phase angle will be $+ \pi/6$ Radians or $+30$ degrees.

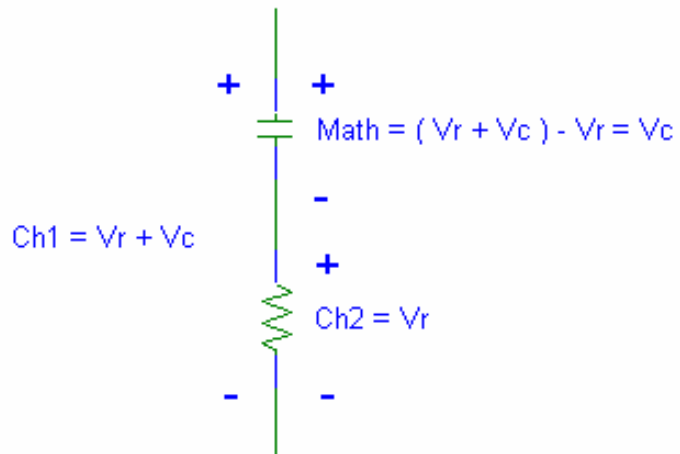
Example: $v(t) = 141.4 \cos(377t - \pi/4)$ Volts

Voltage Phase Angle = $(-\pi/4) * (360/2\pi) = -45$ Degrees.

Voltage Phasor , $V = 100 \angle -45^\circ$ Volts

Viewing series component voltages with the oscilloscope

The oscilloscope can be used to view separate component voltages with the two probes. However, both probes must share a common ground (never float the black lead). Thus, the voltages of two components in series can not be viewed directly with each of the two probes. Should one desire to view the voltage present across the capacitor and the resistor (at the same time) in the figure to the right, the Math function of the oscilloscope must be used. To do this connect the oscilloscope as shown in the figure, press the Math button and use the buttons on the right of the oscilloscope screen to select (Ch1 – Ch2). A red waveform should appear on the screen, this is voltage across the capacitor.



Measuring phase angle differences between series components

If components in a circuit are being excited at the same frequency but peak at different times, there exists a phase angle between the components. The phase angle is the difference (in degrees) between and two points in the component waveforms. Usually this is the positive or negative peak or the x-axis crossing, either with a positive or negative slope.

To measure the phase angle between two series components, the Math function must be used to used as described in “Viewing series component voltages with the oscilloscope.” The menu prompted by the ‘Measure’ button allows the phase angle between two waveforms to be automatically calculated. The red ‘Math’ waveform is available as an option if it is present on the screen.

Resonance

The resonant frequency (f_0) for a RLC series circuit is $f_0 = 1 / 2\pi\sqrt{LC}$. At f_0 , $\omega_0L = 1/\omega_0C$. Thus, the circuit's total reactance ($j\omega L - j/\omega C$) is zero; the current (I) is in phase with the circuit's source voltage (V_s); the circuit's impedance is a minimum, and the circuit's admittance is a maximum.

EQUIPMENT

- Oscilloscope
- Function generator
- Digital multimeter (DMM)
- Resistor, 1 k Ω
- Inductor, 220 mH
- Capacitor, 1.0, μ F

PROCEDURE

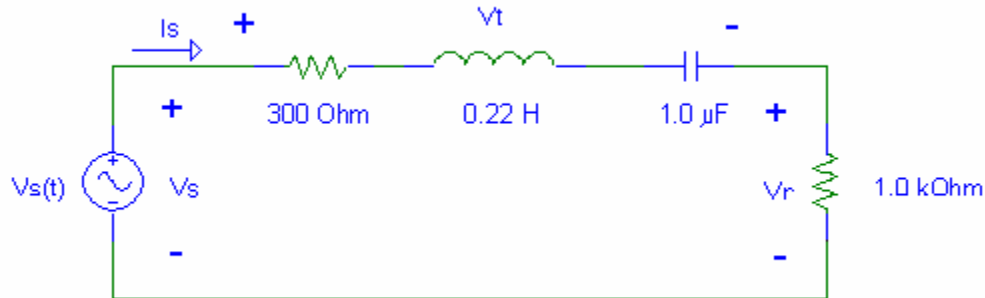


Figure 1: RLC Circuit

- Set the Digital Multimeter (DMM) to read AC Volts and connect it across the output of the Function Generator.
- Select a Sinusoidal output on the Function Generator and adjust its output to read 5 Volts RMS on the DMM.
- Connect the RLC circuit in Figure 1.
- Display V_s and V_r on the Oscilloscope with Channel 1 and 2 respectively. Set the Scope to trigger on a positive slope for Channel 1. Make sure that both channels have Zero Offset (in other words, ground potential will be in the middle of the screen).
- Adjust the frequency of the Function Generator until V_s and V_r are in phase (exactly the same zero crossings).
- Record this frequency as f_o in the middle row of Table 1.

		Measured						Calculated			
Frequency		V_s	$\angle V_s$	V_r	$\angle V_r$	V_t	$\angle V_t$	I_s	Z_s	V_r+V_t	R_s+jX_s
xf_o	Hz	volts	degrees	volts	degrees	volts	degrees	mA	ohms	volts	ohms
0.5 f_o			0								
f_o			0								
2 f_o			0								

Table 1: Experimental and Calculated Data

- Using the Math function of the Oscilloscope, display Ch1 – Ch2 (that is, $V_t = V_s - V_r$).
- Set the oscilloscope to perform the following measurements and record them in the middle row of Table 1:
 - measure the voltage magnitude, V_r , in RMS
 - measure the phase angle of V_r with respect to V_s
 - measure the voltage magnitude, V_t , in RMS
 - measure the phase angle of V_t with respect to V_s
 -
- Use experimental data to make the following calculations and record them in the middle row of Table 1:
 - calculate the current in this series circuit by Ohm's Law:
 $I_r = V_r / (1 \text{ K-Ohm}) \text{ mA}$
 - calculate the total impedance of the circuit by Voltage Divider:
 $Z_s = (1 \text{ K-Ohm}) * V_s / V_r \text{ K-Ohm}$
 - verify KVL by adding the two phasors, $V_r + V_t = V_s$: ****VECTOR ADDITION****
 $[V_r \cos(\text{ang}V_r) + V_t \cos(\text{ang}V_t)] + j [V_r \sin(\text{ang}V_r) + V_t \sin(\text{ang}V_t)]$
 - calculate the rectangular form of Z_s , $R_s + jX_s$:
 $R_s = Z_s \cos(\text{ang}Z_r)$ where $\text{ang}Z_r = -\text{ang}V_r$
 $X_s = Z_s \sin(\text{ang}Z_r)$
- Take a screen capture from the oscilloscope showing the three waveforms, V_s , V_r , V_t .
- Change the frequency of the Function Generator to be $0.5 * f_0$. Repeat the measurements and calculations above to fill in the first line of Table 1.
- Change the frequency of the Function Generator to be $2 * f_0$. Repeat the measurements and calculations above to fill in the last line of Table 1.

IDEAL CALCULATIONS

- Given V_s is a Phasor of 5 Volts RMS and 0 degrees of Phase Angle (chosen as the reference phasor), find V_r as a phasor based on the nominal impedances of the circuit and the frequency of excitation. Fill in Table 2 with the results.

Where:

$$R_s = R_{\text{load}} + R_{\text{inductor}}$$

$$X_s = X_L + X_C = 2 \pi(f)L - 1/[2 \pi(f)C]$$

$$Z_s = R_s + j X_s = (R_s^2 + X_s^2)^{1/2}$$

$$\angle Z_s = \text{atan}(X_s/R_s)$$

$$V_r = (Z_r/Z_s) * V_s$$

Frequency	V_s	R_s	X_s	Z_s	$\angle Z_s$	V_r
$0.5 f_0$	$5 \angle_0 \text{ Vrms}$					
f_0	$5 \angle_0 \text{ Vrms}$					
$2 f_0$	$5 \angle_0 \text{ Vrms}$					

Table 2: Calculated Resistor Voltage

PSPICE ANALYSIS

- Perform an AC sweep analysis of the circuit for Figure 1. In PSPICE Schematics use the VAC model as the voltage source, setting Vmag to 5 Volts. Once the circuit is built, go to ANALYSIS on the menu bar and select SETUP. Enable AC SWEEP ANALYSIS and set it to sweep from 100 Hz to 700 Hz with 100 data points.
- Attach plot of resistor voltage for a frequency range from $0.5f_0$ to $2f_0$

CONCLUSIONS

Normal conclusions based on the experimental process. Make sure to compare your calculated and experimental values of V_r .

SAMPLE CALCULATIONS

Show all of the calculations for $0.5 f_0$ to complete Tables 1 and 2.

