Observations of near-inertial waves in Lake Superior

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The Large Lakes Observatory

• A Wide range of expertise:
  – Geology
  – Biology
  – Chemistry
  – Physics

• Global reach

• A wide range of interests, but a common interest in climate
My lab

- Dynamics and thermodynamics of large lakes
- Wide range of spatial, temporal scales
- Primarily observational, some modeling
- Modern oceanographic equipment
- Lakes around the world
- Lots of student involvement
Student projects

- Bjorn Stolhammer (UG): Correcting slow sensors on autonomous vehicles
- John Brethauer (UG): Descriptions and models of radiatively driven convection below $T_{MD}$
- Dan Titze (PhD, WRS): Role of ice in determining lake thermal structure, (interaction of NIO with sediment)
Core Moorings, 2007-2012
Significant spatial and interannual variability in winter thermal structure
• Surface temperature
• Thermocline depth
Under sustained ice cover, heat loss is inhibited

- **Western**: >99% Reduction
- **Central**: 80% Reduction
- **Eastern**: No Reduction
Energy Distribution

Surf. (Real)

Surf. (Ideal)

Bot. (Real)

Bot. (Ideal)
Energy concentrates around shallow regions in open water
Heating in Lake Superior

- Investigating the lake’s convective warming period
  - Has not been researched extensively in the past
Heating in Lake Superior

- Using fourier techniques for analysis
  - Finding the temperature response to the sun
Heating in Lake Superior

- Current work: Investigating variations in the standard pattern
Temperature convolution

\[
\frac{dT}{dt} = k(T_A - T)
\]

\[
T(t) = T_A * f
\]

\[
f(t) = \begin{cases} 
e^{k(t-t_0)} & t < t_0 \\ 0 & t > t_0 \end{cases}
\]
Outline

• What are near-inertial waves?
• How do we observe them?
• What did we find?
• Why should we (or more to the point, you) care?
Previous work

• Lots of activity in large lakes the 70’s (Marmorino, Mortimer, Schwab, Fee, Rao, etc.)
• Very few observations to work with- severely technologically limited
• Still, nailed down several of the basic features
  – Vertical structure
  – Stratification dependence
• Work done in Kinneret in the last decade... scales are very different
Gravity waves

- Gravity waves represent the response of an density interface to a displacement from equilibrium. Gravity as restoring force
- Energy propagates away from disturbance as waves
- Particles move back and forth in direction of wave, are not displaced over a cycle
The Coriolis force acts perpendicular and in proportion to velocity. $f$ is the Coriolis parameter - about $1 \times 10^{-4} \text{s}^{-1}$.
Poincare waves

- At low frequencies, particle paths become elliptical, approaching circular as their frequency ($\omega$) approaches the Coriolis frequency ($f$) (known as ‘near-inertial waves’)
- Surface (or interface) displacement associated with passage of wave, just as with gravity wave
- Energy concentrated around the inertial frequency ($T \sim 16.1h$)
Interfacial waves

• Waves can exist on any interface between fluids of different densities (e.g. oil and vinegar)
• A stratified lake typically has a strong density interface (the pycnocline) due to temperature stratification
• The speed of waves is proportional to the density difference between the fluids
Observations

• Moorings with current meters in Lake Superior from ~2007 to present
• Several sites; will focus on single sites in the western and eastern basins
• ADCPs (current profilers) and thermistors at a range of depths
Core Moorings, 2007-2012
Acoustic Doppler Current Profiler (ADCP)
Raw velocities, WM (E-W and N-S)
Near-sfc velocities (E-W and N-S)
Temperature, top 80m
Rotary spectrum, currents

(A) Currents

Spectral intensity

10^{-2} \quad 10^{-1} \quad 10^{0} \quad 10^{1}

Cycles per day

Seiche frequency
Inertial frequency
Temperature @13m, WM

Spectral intensity

Cycles per day

Inertial frequency

(E) Temperature (13m)
Wind stress, WM, rotary

Spectral intensity

(CW)

(CCW)

(B) Winds

Inertial frequency

Cycles per day
Water level, Duluth

Diurnal tide

Inertial frequency

Semi-diurnal tide

Seiche

Helmholtz (Harbor) modes

Spectral intensity

Cycles per day

(D) Water level
Raw velocities, WM (E-W and N-S)
\[ w(t, t_0) = \frac{2}{\tau \sqrt{\pi}} e^{-\frac{(t-t_0)^2}{\tau^2}} e^{-j\omega t} \]
\[ P_\pm(t) = \int_{-\infty}^{\infty} w_\pm U dt \]
E-W velocity with wavelet amplitude
CW(red) and CCW (blue) wavelets @16.1h, WM
Estimating the wavelength

• Combine estimates of surface velocity magnitude and interfacial displacement amplitude:

\[ \lambda \sim 2\pi \frac{h_1 |u_1|}{|\eta_2| \omega} \]
Estimating direction

- Direction is a function of the phase difference between interface displacement and surface layer velocity
Too deep-bottom currents weak

Too close to shore

Just right

Backscatter

Depth, m

Distance, km

Too deep-bottom currents weak
Conclusions

• Near-inertial oscillations dominate
• Stratification matters
• Can predict wavelength and direction; direction has interesting behavior
• May play a role in determining sediment remobilization
• Use caution in interpretation of data
  – CTD data
  – Shipboard ADCP data
  – Modeling results
Now What?

- What happens at boundaries?
- What is the distribution of KE at the bottom of the lake?
- What wind conditions are efficient at creating these events?
  - Does the spatial or temporal scale of the wind field matter more?
- How small of a lake can support them?
The Burger number

- Ratio of the horizontal scale of waves (the “Rossby Radius” to basin scale).
- For the “external” (surface) mode:

\[
S = \frac{\sqrt{gH}}{fL} \approx \frac{\sqrt{(9.8 \text{ ms}^{-2})(160 \text{ m})}}{(5 \times 10^5 \text{ m})(1 \times 10^{-4} \text{ s})} \approx 0.8
\]

- External waves can’t “fit” into basin
Internal (Interfacial) mode:

\[
S_I = \frac{\sqrt{g'H'}}{f} = \sqrt{\left(\frac{\Delta \rho}{\rho}\right) \left(\frac{h_1 h_2}{h_1 + h_2}\right)} \frac{\text{L}}{\text{L}}
\]

\[
\approx \sqrt{9.8 \, ms^{-2} \left(\frac{1 \, kgm^{-3}}{1000 \, kgm^{-3}}\right) \left(\frac{(20 \, m)(140 \, m)}{160 \, m}\right)} \approx 0.05
\]