4. Optimality in the Presence of Misclassification

We derived an explicit expression for variance-penalized mean in response-adaptive designs for misclassified dichotomous responses. With restriction on the homogeneous misclassification probabilities, i.e., $\gamma = \gamma_1 = \gamma_2$ and $\delta_1 = \delta_2 = \delta$, VPM is equivalent to

$$E[N_1] = \lambda p_1 - p_2 \text{Var}(N_1),$$

where $p_1 > p_2$, $0 < \delta < 1/2$ and $0 < \lambda < (1 - 2\gamma)^{-1}$. The details and the heterogeneous case can be found in Li and Wang (2012 b).

5. Some Properties

We consider

$$\text{RCMV} = \frac{1}{1 - \gamma - \delta} \left( \frac{1 - 2\gamma}{1 - \gamma - \delta} p_1 - p_2 \right) / \left( \text{Var}(p_1 - p_2) \right).$$

Below is a contour plot of RCMV with a combination of misclassification probabilities $\gamma, \delta \in [0.05, 0.1, 0.2]$, where $\lambda$ is taken as $(1 - 2\gamma)^{-1}/2$.

6. A New Randomization Procedure

We propose a new treatment allocation proportion $p$ that is to be targeted by our design

$$\rho = \frac{1 + \gamma + \delta - \sqrt{\lambda}}{1 + \gamma},$$

where $0 \leq \rho \leq \min (q_1, q_2)/(q_1 + q_2)$. To achieve our design, we implement the doubly adaptive biased coin design (DABC-D) with the allocation function

$$\rho(x, y) = \rho(x, y) = \frac{(1 + \gamma + \delta - \sqrt{\lambda})(1 - q_1) + \gamma}{(1 + \gamma)/(1 + \gamma + \delta - \sqrt{\lambda})}.$$

We consider

$$\text{RCMV} = \frac{1}{1 - \gamma - \delta} \left( \frac{1 - 2\gamma}{1 - \gamma - \delta} p_1 - p_2 \right) / \left( \text{Var}(p_1 - p_2) \right).$$

Below is a contour plot of RCMV with a combination of misclassification probabilities $\gamma, \delta \in [0.05, 0.1, 0.2]$, where $\lambda$ is taken as $(1 - 2\gamma)^{-1}/2$.

Under some regularity conditions, the allocation proportion in our randomization procedure is asymptotically normally distributed

$$N_1 \sim \mathcal{N}(\rho, \sigma^2),$$

where

$$\sigma^2 = \frac{1}{n \rho (1 - \rho) \gamma^2 (1 - \gamma)^2 (1 - 2\gamma)^2}.$$