1. (a) (4 pts) How many distinct permutations can be made from the letters of the word MINNESOTA?

$$k = 181440$$

(b) (4 pts) A committee consists of 3 men and 2 women. Its members can be chosen from a group of 6 men and 5 women. How many different committees are possible?

$$\binom{6}{3} \times \binom{5}{2} = 20 \times 10 = 200$$

(c) (4 pts) A football team plays 7 games with 4 wins, 2 losses and 1 tie. How many distinct ways can this occur?

$$\binom{7}{4} = 105$$

2. (16 pts) A discrete random variable X has the following cumulative distribution function F:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X=x)</td>
<td>0.07</td>
<td>0.26</td>
<td>0.33</td>
<td>0.34</td>
<td>0.42</td>
<td>0.42</td>
<td>0.50</td>
<td>0.95</td>
</tr>
</tbody>
</table>

(a) Determine the value of c such that f(x) is a probability mass function.

$$c = 3$$

(b) Given the student smokes, what is the probability neither parent smokes?

$$0.03/0.18 = 0.16$$

(c) What is the probability that either the student smokes or both parents smoke?

$$0.07 + 0.08 = 0.15$$

(d) Are the event neither parent smokes and the event the student does not smoke independent? Why or why not?

No.

3. (7 pts) Roll a balanced die 3 times. Find the probability of getting a total of three or more in the first two rolls and an even number in the third roll.

$$P(3 \text{ or more in the first two rolls} \cap \text{even in third roll})$$

$$= P(3 \text{ or more in the first two rolls})P(\text{even in third roll})$$

$$= \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

4. (15 pts) Traffic entering an intersection can continue straight ahead, turn right, or turn left. Suppose 65 percent of the traffic continues straight ahead, 20 percent of the traffic turns right, and the remainder of the traffic turns left. Suppose further that a car that continues straight ahead has a probability of 0.004 to be in an accident, a car that turns right has a probability of 0.006 to be in an accident, and a car that turns left has a probability of 0.007 to be in an accident. Find the following probabilities for a randomly selected car entering the intersection.

(a) The car makes a turn.

$$P(\text{right turn}) = 0.20 \quad P(\text{left turn}) = 0.15 \quad P(\text{turn}) = 0.35$$

(b) The car makes an accident.

$$0.65 \times 0.004 + 0.006 + 0.007 = 0.0171$$

(c) The car makes a left turn, given that it has an accident.

$$P(\text{left turn} | \text{accident}) = \frac{0.007}{0.0171} = 0.4099$$

5. (14 pts) A discrete random variable X has the following cumulative distribution function F:

$$F(x) = \begin{cases} 
0, & \text{if } x < 0, \\
0.3, & \text{if } 0 \leq x < 1, \\
0.8, & \text{if } 1 \leq x < 2, \\
1, & \text{otherwise}.
\end{cases}$$

(a) Plot the function F(x).

(b) Determine the probability function of the random variable X, f(x) = P(X=x).

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(c) Calculate the probability P(0.5 < X < 2.5).

$$P(0.5 < X < 2.5) = F(2.5) - F(0.5) = 0.74$$

(d) Calculate the mean E(X).

$$E(X) = \sum_{i=0}^{\infty} x_i f(x_i) = 0.3 \times 0.3 + 0.8 \times 1.0 = 0.77$$

6. (12 pts) A continuous random variable X has a probability density function:

$$f(x) = \begin{cases} 
\frac{c}{x^2}, & \text{if } 0 < x < 1, \\
0, & \text{otherwise}.
\end{cases}$$

(a) Determine the value of c such that f(x) is a probability density function.

$$1 = \int_{0}^{1} \frac{c}{x^2} \, dx = c$$

(b) Find P(0.2 < X < 0.6).

$$P(0.2 < X < 0.6) = \int_{0.2}^{0.6} \frac{c}{x^2} \, dx = 0.288$$

(c) Evaluate E(X^2).

$$E(X^2) = \int_{0}^{1} x^2 \cdot \frac{c}{x^2} \, dx = c$$

(d) Are X and Y independent? Why?

Yes, since f(x,y) = g(x)h(y).

7. (24 pts) Two discrete random variables X and Y have a joint probability function f(x,y) = P(X=x,Y=y) = \begin{cases} 
(2x+y)^3 & \text{if } x=1, y=1; \\
11 & \text{if } x=2, y=1; \\
72 & \text{if } x=3, y=1; \\
0 & \text{otherwise},
\end{cases}

(a) Fill in the following table with proper probabilities:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>0.04</td>
<td>0.08</td>
<td>0.12</td>
</tr>
<tr>
<td>1</td>
<td>0.06</td>
<td>0.12</td>
<td>0.18</td>
<td>0.24</td>
</tr>
<tr>
<td>2</td>
<td>0.04</td>
<td>0.08</td>
<td>0.12</td>
<td>0.16</td>
</tr>
<tr>
<td>3</td>
<td>0.06</td>
<td>0.12</td>
<td>0.18</td>
<td>0.24</td>
</tr>
</tbody>
</table>

(b) Find the marginal distributions for X and Y.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>g(x)</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.30</td>
</tr>
</tbody>
</table>

(c) Find the probability P(X|Y).

P(X=1|Y=2) = 0.12/0.60 = 0.20

(d) What is the conditional probability of X=1, given Y=2?

P(Y=2|X=1) = 0.12/0.60 = 0.20

(e) Calculate E(X|Y).

1(0.08) + 2(0.12) + 3(0.12) = 0.36

(f) Are X and Y independent? Why?

Yes, since f(x,y) = g(x)h(y) for all x, y.