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STAT 3611♦Spring 2005

Introduction to Probability and Statistics

Midterm Exam 1, Friday, Feb 25, 2005

You may use a **calculator** in this exam. **Check the last page for two formulae**.
(This test consists of 4 pages)

1. **(a)** (4 pts) How many distinct permutations can be made from the letters of the word **MINNESOTA**?

$$\frac{9!}{2!}=181440$$

(b) (4 pts) A committee consists of 3 men and 2 women. Its members can be chosen from a group of 6 men and 5 women. How many different committees are possible?

$$\binom{6}{3}\binom{5}{2}=20*10=200$$

(c) (4 pts) A football team plays 7 games with 4 wins, 2 losses and 1 tie. How many distinct ways can this occur?

$$\frac{7!}{4!2!1!}=105$$

2. (16 pts) A study of the effect of parents' smoking habits on the smoking habits of students in a high school produced the following table of proportions.

Parents	Student smokes	Student does not smoke	Total
Both parents smoke	0.07	0.26	0.33
One parent smokes	0.08	0.34	0.42
Neither parent smokes	0.03	0.22	0.25
Total	0.18	0.82	1.00

Suppose a student is randomly selected from this population.

a. What is the probability the student smokes and at least one parent of the student smokes?

0.07+0.08=0.15

b. Given the student smokes, what is the probability neither parent smokes?

0.03/0.18=1/6

c. What is the probability that either the student smokes or both parents smoke?

0.07+0.26+0.08+0.03=0.44 or 0.18+0.33-0.07=0.44

d. Are the event neither parent smokes and the event the student does not smoke independent? Why or why not?

No. 0.22≠0.82*0.25

3. (7 pts) Roll a balanced die 3 times. Find the probability of getting a total of three or more in the first two rolls and an even number in the third roll.

P(a total of three or more in the first two rolls)P(an even number in the third roll)
=(1-P(total 2 in the first two rolls)P(even in third roll)

$$=(1-\frac{1}{36})(\frac{3}{6})=\frac{35}{72}$$

4. (15 pts) Traffic entering an intersection can continue straight ahead, turn right, or turn left. Suppose 65 percent of the traffic continues straight ahead, 20 percent of the traffic turns right, and the remainder of the traffic turns left. Suppose further that a car that continues straight ahead has a probability of 0.0004 to be in an accident, a car that turns right has a probability of 0.0036 to be in an accident, and a car that turns left has a probability of 0.0075 to be in an accident. Find the following probabilities for a randomly selected car entering the intersection

a. The car makes a turn.
P(right turn)=0.20 P(left turn)=0.15 **P(turn)=0.35**

b. The car has an accident.

0.65*0.0004+0.15*0.0075+0.20*0.0036=**0.002105**

c. The car was making a left turn, given that it has an accident.

0.15*0.0075/0.002105=**0.5344**

5. (14 pts) A discrete random variable X has the following cumulative distribution function F:

$$F(x)=\begin{cases}0, & x<0,\\ 0.3, & 0\le x<1,\\ 0.4, & 1\le x<2,\\ 0.8, & 2\le x<3,\\ 1, & x\ge 3.\end{cases}$$

(a). Plot the function F(x).

(b). Determine the probability function of the random variable X, f(x)=P(X=x).

x	=0	1	2	3
f(x)	=0.3	0.1	0.4	0.2

(c). Calculate the probability P(0.5<X<2.5).

f(1)+f(2)=0.1+0.4=0.5

(d). Calculate the mean $E(X^3)$.

$$EX^3=0^3(0.3)+1^3(0.1)+2^3(0.4)+3^3(0.2)=8.7$$

6. (12 pts) A continuous random variable X has a probability density *f*(x):

$$f(x)=\begin{cases}cx^2, & 0<x<1,\\ 0, & otherwise.\end{cases}$$

(1). Determine the value of c such that f(x) is a probability density.

$$1=c\int_0^1 x^2dx=\frac{cx^3}{3}\Big|_0^1=c/3 \rightarrow c=3$$

(2). Find P(0.2<X<0.6).

$$\int_{0.2}^{0.6}3x^2dx=x^3\Big|_{0.2}^{0.6}=0.208$$

(3). Evaluate $E(X^2)$.

$$\int_0^1 x^2 3x^2dx=\frac{3x^5}{5}\Big|_0^1=\frac{3}{5}$$

7. (24 pts) Two **discrete** random variables X and Y have a joint probability function $f(x,y)=P(X=x,Y=y)=\frac{(x+1)(y+1)}{50}$, x=0,1,2,3; y=1,2.

(1). Fill in the following table with proper probabilities:

f(x,y)	x=0	1	2	3
y=1	0.04	0.08	0.12	0.16
2	0.06	0.12	0.18	0.24

(2) Give the marginal distributions for X and Y.

x	0	1	2	3
g(x)	0.10	0.20	0.30	0.40

y	1	2
h(y)	0.40	0.60

(3) Find the probability P(X≠Y).

1-f(1,1)-f(2,2)=1-0.08-0.18=**0.74**

(4) What is the conditional probability of X=1, given Y=2?

P(X=1|Y=2)=0.12/0.60=0.20

(5) Calculate E(XY).

(1)(1)(0.08)+(1)(2)(0.12)+(2)(1)(0.12)+(2)(2)(0.18)+(3)(1)(0.16)+(3)(2)(0.24)
=**3.2**

(6) Are X and Y are independent? Why?

Yes, verify f(x,y)=g(x)h(y) for all x, y

-------------------**end of test**---

Theorem (Bayes's Rule) . Let $\{B_1, B_2, \ldots, B_k\}$ constitute a partition of the sample space S. Then for any event A of S with P(A)>0,

$$P(B_r\mid A)=\frac{P(B_r\cap A)}{\sum_{i=1}^k P(B_i\cap A)}=\frac{P(B_r)P(A\mid B_r)}{\sum_{i=1}^k P(B_i)P(A\mid B_i)} \quad \text{for } r=1, 2, \ldots k.$$

Theorem (theorem of total probability) Let $\{B_1, B_2, \ldots, B_k\}$ constitute a partition of the sample space S such that $P(B_i)>0$ for i=1, …, k. Then for any event A of S

$$P(A)=\sum_{i=1}^k P(B_i\cap A)=\sum_{i=1}^k P(B_i)P(A\mid B_i).$$