Math 3298 Exam 1
NAME: **SCORE:**

1. Given $\mathbf{a} = \langle 2, 0, 1\rangle,\ \mathbf{b} = \langle 1, 2, 3\rangle$. Find (i) $2\mathbf{a} + \mathbf{b}$. (ii) $\mathbf{a} \cdot \mathbf{b}$. (iii) $\mathbf{a} \times \mathbf{b}$, (iv) $\text{Proj}_{\mathbf{a}}\mathbf{b}$, (v) $\text{Comp}_{\mathbf{a}}\mathbf{b}$

2. Given three points $A(0, 0, 1),\ B(1, 1, 1)\ C(-1, -1, -1)$. Find vectors $\vec{AB},\ \vec{AC}$. Show that these two vectors are perpendicular.

3. Given two points $(4, -1, 2), (1, 1, 5)$. Find the parametric equation of the line passing these two points.

4. Find the plane equation that passing through the points $A(2, 1, 1), B(-1, -1, 10), C(1, 3, -4)$.

5. Find the plane equation passing the point $(2, 0, 4)$ with normal direction $\langle 2, -4, -3 \rangle$. Moreover, find the angle between this plane and the plane in last problem.

6. Find the volume of the parallelepiped spanned by three vectors $\langle 1, 1, 0 \rangle, \langle 2, 1, 0 \rangle, \langle 1, 1, 1 \rangle$.

7. Make a rough sketch of the quadratic surfaces, and give their names (i) $4x^2 + 9y^2 + z = 0$. (ii) $-x^2 + 4y^2 - z^2 = 4$.

8. Find a vector function $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ for the curve of the intersection of the cylinder $x^2 + y^2 = 16$ and the plane $x + z = 5$. (hint: let $x = t$ be the parameter. Express y, z in terms of t).

9. A particle moves with position function $\mathbf{r}(t) = \langle t, 2t, t^2 \rangle$. Find its velocity, speed, and acceleration.

10. A particle starts at the origin with initial velocity $\mathbf{v}(0) = \langle 1, -1, 3\rangle$. Its acceleration is $\mathbf{a}(t) = \langle 6t, 12t^2, -6t\rangle$. Find its position vector $\mathbf{r}(t)$.

11. Find the curvature of the ellipse $\mathbf{r}(t) = \langle 3\cos t, 4\sin t, 0\rangle$ at the point $(0, 4, 0)$. Find $\mathbf{T}(t), \mathbf{N}(t), \mathbf{B}(t)$ at $t = \pi/2$.

12. Suppose $\mathbf{a} \neq \mathbf{0}$. If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, can we claim that $\mathbf{b} = \mathbf{c}$? If Yes, prove it. If no, give an example to support your conclusion.