Math 3298 Exam III

NAME: **SCORE:**

1. Find a parametric equation for the curve $y = x^2$ starting from $(0,0)$ and ending at $(1,1)$. Specify the parameter region.

2. Find a parametric equation for the surface which is the part of the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $z = \sqrt{x^2 + y^2}$. Specify the parameter region.

3. Given a parametric surface

$$\mathbf{r}(u,v) = \langle u\cos v, u\sin v, \sin u\rangle, \qquad u \in [-\pi, \pi], \quad v \in [0, 2\pi].$$

Find its Cartesian equation.

4. Given a vector field $\mathbf{F} = \langle p(x,y,z), Q(x,y,z), R(x,y,z)\rangle$. Define curl$\mathbf{F}$ and div$\mathbf{F}$. What is the definition of conservative vector field?

5. Compute the line integral

$$\int_C yz \cos x ds, \qquad C : x = t,\ y = 3\cos t,\ z = 3\sin t,\ 0 \leq t \leq \pi.$$

6. Use Green's Theorem to evaluate

$$\int_C \sqrt{1 + x^3} dx + 2xy dy$$

where C is the triangle with vertices $(0, 0), (1, 0)$, and $(1, 3)$.

7. Suppose that you know that $f(x,y) = e^x + x^2y^2$ and

$$\nabla f = \mathbf{F} = \langle e^x + 2xy^2, 2x^2y \rangle.$$

Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C : is $\mathbf{r}(t) = \langle t + \sin \pi t, 2t + \cos \pi t \rangle, 0 \le t \le 1$.

8. Evaluate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F}(x,y,z) = \langle xz, -2y, 3x \rangle, \qquad S : \mathbf{r}(u,v) = \langle 2\sin u \cos v, 2\sin u \sin v, 2\cos u \rangle$$

and $0 \le u \le \pi$, and $0 \le v \le 2\pi$.

9. Given $\mathbf{F} = \langle x^2, y^2, z^2 \rangle$, and surface S is the part of the paraboloid $z = 1 - x^2 - y^2$ above the XY-plane. Let C be the boundary curve of S. Verify the Stokes' Theorem for this problem.