

Stat 3611 Quiz 2
NAME:

1. A certain federal agency employs three consulting firms (A, B, and C) with probabilities 0.40, 0.35, and 0.25, respectively. From the past experience it is known that the probability of cost overruns for the firms are 0.05, 0.03, and 0.15, respectively. Suppose a cost overrun is experienced by the agency.

(a) What is the probability that the consulting firm involved is C?

$$P(A) = 0.4, P(O|A) = 0.05$$

(b) What is the probability that the consulting firm involved is A?

$$P(B) = 0.35, P(O|B) = 0.03$$

Let A, B, C = firm A, B, C hired, respectively

$$P(C) = 0.25, P(O|C) = 0.15$$

O = cost overrun

$$(a) P(C|O) = \frac{P(C)P(O|C)}{P(A)P(O|A) + P(B)P(O|B) + P(C)P(O|C)} = \frac{(0.25)(0.15)}{(0.4)(0.05) + (0.35)(0.03) + (0.25)(0.15)} = \boxed{0.5515}$$

$$(b) P(A|O) = \frac{P(A)P(O|A)}{P(A)P(O|A) + P(B)P(O|B) + P(C)P(O|C)} = \boxed{0.294}$$

2. An overseas shipment of 5 foreign automobiles contains 2 that have slight paint blemishes. If an agency receives 3 of these automobiles at random,

(a) list all the elements of the sample space S , using letters B and N for blemished and nonblemished, respectively.

(b) Find the probability distribution function $f(x)$ for the random variable X .

X = # of car blemished, 0, 1, 2

$$(a) \{NNN, NNB, \cancel{NNB}, \cancel{NBN}, \cancel{BNN}, \cancel{BNN}, NBB\}$$

$$(b) \begin{array}{cccc} x & 0 & 1 & 2 \\ f(x) & \frac{1}{10} & \frac{6}{10} & \frac{3}{10} \end{array}$$

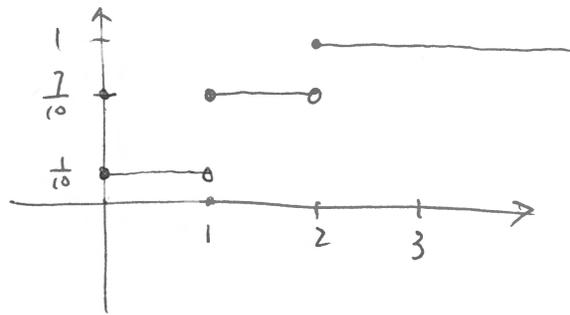
$$f(0) = \frac{\binom{3}{3}\binom{2}{0}}{\binom{5}{3}} = \frac{1}{10}$$

$$f(1) = \frac{\binom{3}{2}\binom{2}{1}}{\binom{5}{3}} = \frac{6}{10}$$

$$f(2) = \frac{\binom{3}{1}\binom{2}{2}}{\binom{5}{3}} = \frac{3}{10}$$

3. Find the cumulative distribution function $F(x)$ for last problem, and plot its graph.

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{10} & 0 \leq x < 1 \\ \frac{2}{10} & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$



4. The proportion of the budget for a certain type of industrial company that is allotted to environmental and pollution control is represented by a random variable X . A data collection project determines that the distribution of these proportions is given by

$$f(x) = \begin{cases} 5(1-x)^4, & 0 \leq x \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Verify that $f(x)$ is a valid density function.
 (b) What is the probability that a company chosen at random expends less than 10% of its budget on environmental and pollution control?
 (c) Find the CDF.

(a) $1 = \int_{-\infty}^{\infty} f(x) dx = \int_0^1 5(1-x)^4 dx = -\left. (1-x)^5 \right|_0^1 = -0 + 1 = 1$

and $f(x) \geq 0$. Thus, it is a valid PDF.

(b) $P(X \leq 0.1) = \int_0^{0.1} 5(1-x)^4 dx = -\left. (1-x)^5 \right|_0^{0.1} = -(0.9)^5 + 1 \approx 0.41$

(c) $F(x) = \int_{-\infty}^x f(t) dt = -\left. (1-t)^5 \right|_0^x = -(1-x)^5 + 1 \quad \text{for } 0 \leq x < 1$

$F(x) = 0 \quad \text{for } x \leq 0, \quad F(x) = 1 \quad \text{for } x > 1$

SCORE:

$$F(x) = \begin{cases} 0 & x \leq 0 \\ 1 - (1-x)^5 & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$