Stat 3611 Quiz 3
NAME:

1. A construction company employs two sales engineers. Engineer 1 does the work of estimating cost for 70% of jobs bid by the company. Engineer 2 does the work of estimating cost for 30% of jobs bid by the company. It is known that the error rate for engineer 1 is 0.02, for engineer 2 is 0.04. Suppose a bid arrives and a serious error occurs in estimating cost. Which engineer is more likely did the work?

E_1 – engineer 1 does the work

E_2 – ..... 2 does ... .

A – error occurs

$P(E_1)=0.7$. $P(E_2=0.3)$. $P(A|E_1)=0.02$. $P(A|E_2)=0.04$

$$P(E_1|A)=\frac{(0.7)(0.02)}{(0.7)\cdot(0.02)+(0.3)(0.04)}$$

$$P(E_2|A)=\frac{(0.3)(0.04)}{(0.7)(0.02)+(0.3)\cdot(0.04)}$$

$\Rightarrow P(E_1|A)>P(E_2|A)$

Answer: **Engineer 1**

2. Three cards are drawn without replacement from the 12 face cards (J, Q, K). Let X be the number of kings selected and Y the number of jacks.

   (a) Find the joint probability distribution of X and Y.

   (b) Find marginal density function for X and Y.

(a)

$$P(X=x, Y=y)=\frac{\binom{4}{x}\binom{4}{y}\binom{4}{3-x-y}}{\binom{12}{3}}=\frac{\binom{4}{x}\binom{4}{y}\binom{4}{3-x-y}}{220}$$

f(x,y)	X: 0	1	2	3
y = 0	4/220	24/220	24/220	4/220
1	24/220	64/220	24/220	0
2	24/220	24/220	0	0
3	4/220	0	0	0

(b)

X	0	1	2	3
g(x)	56/220	112/220	48/220	4/220

y	0	1	2	3
h(y)	56/220	112/220	48/220	4/220

3. (a) Find $P[(X,Y) \in A]$ where A is the region given by $x + y \geq 2$ in the last problem.
   (b) Find $P(X = 1 \mid Y = 1)$.

(a) $P(X+Y \geq 2) = 1 - P(X+Y < 2) = 1 - P(0,0) - P(0,1)$

$-P(1,0) = \frac{168}{220}$

(b) $P(X=1 \mid Y=1) = \frac{f(1,1)}{h(1)} = \frac{\frac{64}{220}}{\frac{112}{220}} = \frac{64}{112} = \frac{4}{7}$

4. Let X and Y denote the lenths of life, in years, of two components in an electronic system. If the joint density function of these variables is

$$f(x,y) = \begin{cases} e^{-x}e^{-y}, & x > 0, y > 0 \\ 0, & \text{elsewhere} \end{cases}$$

(a) find marginal density for X and Y.
(b) find $P(0 < X < 1 \mid 0 < Y < 2)$

(a) $g(x) = \int_0^\infty e^{-x}e^{-y}\,dy = -e^{-x} \cdot e^{-y}\Big|_0^\infty = \boxed{e^{-x}}$ $x > 0$

$h(y) = \int_0^\infty e^{-x}e^{-y}\,dx = -e^{-y} \cdot e^{-x}\Big|_0^\infty = \boxed{e^{-y}}$ $y > 0$

(b) $P(0<X<1 \mid 0<Y<2) = \frac{P(0<X<1,\ 0<Y<2)}{P(0<Y<2)}$

$= \frac{\int_0^1 \int_0^2 e^{-x}e^{-y}\,dy\,dx}{\int_0^2 e^{-y}\,dy} = \int_0^1 e^{-x}\,dx = \boxed{1 - e^{-1}}$

SCORE: