

Stat 3611 Quiz 4

NAME:

1. Suppose that you buy a share of stock, then sell it 30 days later. If the probability of making a profit of \$250, \$150, \$0, - \$150 are 0.22, 0.36, 0.28 and 0.14, respectively. What is the expected gain? What is the variance? Let X be the profit random variable.

X	250	150	0	-150
$f(X)$	0.22	0.36	0.28	0.14
X^2	62500	22500	0	22500

$$E(X) = (250)(0.22) + (150)(0.36) + (0)(0.28) + (-150)(0.14) \neq$$

$$= 55 + 33 = \boxed{88}$$

$$\text{Var}(X) = E(X^2) - (E(X))^2 = (62500)(0.22) + (22500)(0.36) + (22500)(0.14) - 88^2$$

$$= 13750 + 11250 - \cancel{26250} = 7744$$

$$= \boxed{17256}$$

2. In a gambling game, a woman is paid \$3 if she draws a jack or a queen and \$5 if she draws a king or an ace from an ordinary deck of 52 playing cards. If she draws any other card, she loses. How much should she pay if the game is fair?

X — gain from each play

X	3	5	k
$f(x)$	$\frac{8}{52}$	$\frac{8}{52}$	$\frac{36}{52}$

$$\left\{ \begin{array}{l} P(X=3) = \frac{8}{52} \\ P(X=5) = \frac{8}{52} \\ P(X=k) = \frac{36}{52} \end{array} \right.$$

Game is fair $\Leftrightarrow E(X) = 0$

$$E(X) = 3 \cdot \frac{8}{52} + 5 \cdot \frac{8}{52} + k \cdot \frac{36}{52} = \frac{64 + 36k}{52} = 0$$

Then $k = -\frac{64}{36} = -\frac{16}{9}$.

She should pay $\boxed{\$ \frac{16}{9}}$

3. A continuous random variable X has the density function

$$f(x) = \begin{cases} e^{-x}, & x > 0, \\ 0, & \text{elsewhere} \end{cases}$$

Find the mean of $g(X) = e^{-2X}$.

$$\begin{aligned} E(g(x)) &= \int_{-\infty}^{\infty} g(x) f(x) dx = \int_0^{\infty} e^{-2x} \cdot e^{-x} dx \\ &= \int_0^{\infty} e^{-3x} dx = -\frac{1}{3} e^{-3x} \Big|_0^{\infty} = -\frac{1}{3} (0 - 1) = \boxed{\frac{1}{3}} \end{aligned}$$

4. Find the variance of $g(X) = 2X + 1$ for the X given in problem 1.

$$\begin{aligned} E(2X+1) &= E(2X) + E(1) = 2E(X) + 1 \\ &= 2 \cdot 88 + 1 = 177 \end{aligned}$$

$$\text{Var}(2X+1) = 4 \text{Var}(X) = \boxed{69024}$$

$$\text{or } \sum_x (g(x) - \mu_{g(x)})^2 f(x) = (501)^2 \cdot (0.22) + (301)^2 \cdot (0.36) + 1^2 \cdot 0.28 + (-299)^2 \cdot (0.14) - (177)^2 = 69024$$

5. Find conditional probability $P(X=2|Y=3)$ from the given joint PMF

$f(x, y)$		x		
		2	4	
y	1	0.10	0.15	0.25
	3	0.20	0.30	0.5
	5	0.10	0.15	0.25

$$P(X=2|Y=3) = \frac{P(X=2, Y=3)}{P(Y=3)} = \frac{0.2}{0.5} = \boxed{0.4}$$