Problem 1. Consider the following augmented matrices. If the corresponding linear system has a solution, find the solution and determine if it is unique, otherwise state why no solution exists.

\[ a. \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix} \]

\[ b. \begin{bmatrix} 1 & 2 & 1 \\ -1 & 2 & 3 \\ -1 & 3 & 4 \end{bmatrix} \]

\[ c. \begin{bmatrix} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 5 & -8 & 7 & 1 \end{bmatrix} \]

Problem 2. Consider the system of equations

\[-8x_1 + 4x_2 + 0x_3 = -1 \]
\[ 4x_1 - 8x_2 + 4x_3 = -2 \]
\[ 0x_1 + 4x_2 - 8x_3 = -3 \]

a. Write the system as an augmented matrix

b. Use row operations to produce an augmented matrix in echelon form (the coefficient matrix should be upper triangular).

c. Use back substitution, or fully reduce the augmented matrix, in order to solve the system of equations for \( x_1, x_2, x_3. \)

Problem 3. Let \( u = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}. \)

a. Draw the parallelogram whose vertices are 0, \( u, v, \) and \( u + v. \)

b. Let \( b = \begin{bmatrix} 5 \\ 0 \end{bmatrix}. \) Find \( x_1 \) and \( x_2 \) such that \( x_1 u + x_2 v = b. \)

c. Give a graphical justification for why every vector \( b \in \mathbb{R}^2 \) can be written as a linear combination of \( u \) and \( v. \)
Problem 4. Consider the following vector equation

\[
\begin{bmatrix}
1 \\
2 \\
5
\end{bmatrix}
x_1 + \begin{bmatrix}
2 \\
-5 \\
-6
\end{bmatrix}x_2 = \begin{bmatrix}
7 \\
-4 \\
3
\end{bmatrix}
\]

a. Write the vector equation as a system of linear equations.

b. Write the system of linear equations as an augmented matrix.

c. If the system has a solution, find the solution and determine if it is unique, otherwise state why no solution exists.

Problem 5. Given vectors \(a_1, \ldots, a_k \in \mathbb{R}^n\) provide a definition of the following:

a. a linear combination of the vectors \(a_1, \ldots, a_k\),

b. the spanning set of the vectors \(a_1, \ldots, a_k\).