[15] 1. Find the equation of the plane that contains both of the intersecting lines \( \vec{r}_1(t) = (1 + t, 2 - 3t, 5 - 4t) \) and \( \vec{r}_2(s) = (1 - 2s, 2 + s, 5 - 3s) \).

\[
\begin{vmatrix}
2 & 1 & 1 \\
1 & -3 & -4 \\
-2 & 1 & 3
\end{vmatrix} = (13, 11, -5), \quad \text{so the plane is}
\]

\[
13(x-1) + 11(y-2) - 5(z-5) = 0
\]

or
\[
13x + 11y - 5z = 10.
\]

[15] 2. Find parametric equations for the line that contains the point \((-2, 1, 3)\) and is perpendicular to the plane \(6x - y - 3z = 10\).
3. For the curve \( \mathbf{r}(t) = (t - 3, \sqrt{t}) \), do the following:

a) Find \( \mathbf{r}'(t) \).
\[
\mathbf{r}'(t) = \langle 1, \frac{1}{2\sqrt{t}} \rangle.
\]

b) Find the \( t \) value for which \( \mathbf{r}(t) \) and \( \mathbf{r}'(t) \) are orthogonal. That is, find \( t \) such that \( \mathbf{r}(t) \cdot \mathbf{r}'(t) = 0 \)
\[
\mathbf{r}(t) \cdot \mathbf{r}'(t) = \langle t-3, \sqrt{t} \rangle \cdot \langle 1, \frac{1}{2\sqrt{t}} \rangle = \frac{t}{2} = t - \frac{5}{2}.
\]
This is zero when \( t = \frac{5}{2} \). 

4. Suppose a particle's acceleration is \( \mathbf{a}(t) = (3, t^2, e^{-t}) \), and the particle's initial velocity is \( \mathbf{v}(0) = (2, 1, 0) \).

a) Find the particle's velocity function \( \mathbf{v}(t) \).
\[
\mathbf{v} = \int \mathbf{a} \, dt = \langle 3t + A, \frac{1}{3}t^3 + B, -e^{-t} + C \rangle.
\]
\[
\mathbf{v}(0) = \langle 2, 1, 0 \rangle \Rightarrow \langle A, B, -1 + C \rangle = \langle 2, 1, 0 \rangle
\]
\[
\Rightarrow A = 2, \quad B = 1, \quad C = 1.
\]
\[
\mathbf{v}(t) = \langle 3t + 2, \frac{1}{3}t^3 + 1, -e^{-t} + 1 \rangle.
\]

b) Set up an integral whose value would give the distance travelled by the particle from \( t = 0 \) to \( t = 1 \).
\[
\int_0^1 ||\mathbf{v}(t)|| \, dt = \int_0^1 \sqrt{(3t+2)^2 + \left(\frac{1}{3}t^3 + 1\right)^2 + (-e^{-t} + 1)^2} \, dt.
\]
5. For the surface \( \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} = 1 \), do the following:

a) Below, the surface's trace in the \( xy \)-plane has been found and graphed. Find and graph the surface's traces in the \( zz \)- and \( yz \)-planes.

\[\text{xy trace: set } z = 0, \text{ get } x + \frac{y^2}{9} = 1, \text{ so } x = 1 - \frac{y^2}{9};\]

\[\text{xz trace: set } y = 0, \text{ get } x = 1 - \frac{z^2}{4};\]

\[\text{yz trace: set } x = 0, \text{ get } \frac{y^2}{9} + \frac{z^2}{4} = 1;\]

b) Which graph below best represents this surface? (circle your answer)

6. For the function \( f(x, y) = \sqrt{x^2 + y} \), graph the level curves (a.k.a. contours) \( f(x, y) = 0, f(x, y) = 1, f(x, y) = 2, f(x, y) = 3 \) in the window provided. Label each curve with its \( z \)-level.

\[\sqrt{x^2 + y} = 0 \quad \Rightarrow \quad y = -x^2\]

\[\sqrt{x^2 + y} = 1 \quad \Rightarrow \quad y = 1 - x^2\]

\[\sqrt{x^2 + y} = 2 \quad \Rightarrow \quad y = 4 - x^2\]

\[\sqrt{x^2 + y} = 3 \quad \Rightarrow \quad y = 9 - x^2\]
[15] 7. Let function $h$ be defined like this: 

$$h(x, y) = \begin{cases} \frac{3xy}{x^2 + 4y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

should be $(0, 0)$.

[10] a) Prove that $\lim_{(x,y) \to (0,0)} \frac{3xy}{x^2 + 4y^2}$ does not exist.

Along $x$-axis, \[ \frac{3xy}{x^2 + 4y^2} = \frac{3x \cdot 0}{x^2 + 4 \cdot 0^2} = \frac{0}{x^2} \to 0. \]

Along $y = x$, \[ \frac{3xy}{x^2 + 4y^2} = \frac{3x^2}{x^2 + 4x^2} = \frac{3x^2}{5x^2} \to \frac{3}{5}. \]

So by the two path test, this limit DNE.

[5] b) At which points in $\mathbb{R}^2$ is $h$ continuous? At which points in $\mathbb{R}^2$ is $h$ discontinuous? Explain.

$h$ is continuous at all $(x, y) \neq (0, 0)$, because at these points $h$ coincides with $\frac{3xy}{x^2 + 4y^2}$, which is known to be cont. for all $(x, y) \neq (0, 0)$.

$h$ is discontinuous at $(0, 0)$, because $\lim_{(x,y) \to (0,0)} h(x,y)$ is the limit from part (a), which DNE.

[15] 8. Find $f_x(x, y)$ and $f_y(x, y)$ for the function $f(x, y) = x^2y^3 + 5x - \frac{x}{y^2}$.

$$f_x = \frac{\partial f}{\partial x} = \sqrt{2x^2y^3 + 5 - \frac{1}{y^2}}$$

$$f_y = \frac{\partial f}{\partial y} = \sqrt{3x^2y^2 + \frac{2x}{y^3}}$$