1. Let $M = (Q, \Sigma, q_0, \delta)$ be an FA. We say that a state $q \in Q$ is **doomed** in $M$ if there is no string $x \in \Sigma^*$ s.t. $\delta^*(q, x) \in A$. Prove that, for all $q \in Q$ and $a \in \Sigma$, if $q$ is doomed in $M$, then so is $\delta(q, a)$.

2. Let $M = (Q, \Sigma, q_0, \delta)$ be an FA with at least one state that is doomed. (See definition in previous problem.) Let $D$ be the set of all doomed states in $M$, and let $d$ be a doomed state in $M$ s.t. $q_0 \notin D - \{d\}$. (So if $q_0$ is doomed, then $d$ is $q_0$.) Consider the FA $M_1 = (\{d\} \cup (Q - D), \Sigma, q_0, A, \delta_1)$ where, for all $q \in \{d\} \cup (Q - D)$ and all $a \in \Sigma$,

$$\delta_1(q, a) = \begin{cases} d & \delta(q, a) \in D \\ \delta(q, a) & \text{otherwise} \end{cases}$$

It is possible to show that for all $x \in \Sigma^*$,

- if $\delta^*(q_0, x) \in D$, then $\delta_1^*(q_0, x) = d$,
- if $\delta^*(q_0, x) \notin D$, then $\delta_1^*(q_0, x) = \delta^*(q_0, x)$.

Call this fact the **Main Lemma**. Use the Main Lemma to prove that $L(M) = L(M_1)$.

3. For the NFA-$\Lambda$ shown below,

find the following.

(a) $\Lambda(\emptyset) =$
(b) $\Lambda(\{1\}) =$
(c) $\delta(1, \Lambda) =$
(d) $\delta^*(1, \Lambda) =$
(e) $\delta(1, b) =$
(f) $\delta^*(1, b) =$
(g) $\delta^*(1, ba) =$
(h) $\delta^*(1, baa) =$

4. Say whether the two NFA-$\Lambda$'s shown below accept the same language.
5. Say whether the two NFA-Λ's shown below accept the same language.

6. Using the construction from Theorem 4.2, reduce the following NFA-Λ to an NFA.

7. Use the construction from the proof of Theorem 4.4 (half of Kleene's Theorem) to obtain an NFA-Λ for the language corresponding to the regular expression \((ab + \Lambda)(c + d)^*\).

8. For any NFA-Λ \(M = (Q, \Sigma, q_0, A, \delta)\), let \(M_\Lambda\) denote the NFA-Λ obtained by adding \(\Lambda\)-transitions from each state in \(A\) to \(q_0\), and making \(q_0\) the lone accepting state. Find an NFA-Λ \(M\) s.t. \(L(M_\Lambda) \not\subseteq L(M)^*\), and identify a string that shows this.

9. For any NFA-Λ \(M = (Q, \Sigma, q_0, A, \delta)\), let \(M_\Lambda\) denote the NFA-Λ obtained by adding \(\Lambda\)-transitions from each state in \(A\) to \(q_0\), and from \(q_0\) to each state in \(A\). Find an NFA-Λ \(M\) s.t. \(L(M)^* \not\subseteq L(M_\Lambda)\), and identify a string that shows this.

10. Consider the following alternative to the construction used in the proof of Theorem 4.4 of an NFA-Λ \(M_u\) to accept \(L(M_1) \cup L(M_2)\), given NFA-Λ's \(M_1\) and \(M_2\): Instead of a new initial state \(q_u\) and \(\Lambda\)-transitions from \(q_u\) to \(q_1\) and \(q_2\), make \(q_1\) the initial state of the new NFA-Λ, and add a \(\Lambda\)-transition from \(q_1\) to \(q_2\). Either prove that this works in general, or show that it does not always work.

11. Prove by structural induction that the \(\Lambda\)-closure function \(\Lambda : 2^Q \rightarrow 2^Q\) used in the definition of \(\delta^*\) for NFA-Λ's is monotone: that is, prove that, for any \(S, T \subseteq Q\), if \(S \subseteq T\), then \(\Lambda(S) \subseteq \Lambda(T)\). So you will assume \(S \subseteq T \subseteq Q\), and then use structural induction on the recursive definition of \(\Lambda(S)\). (As usual, be explicit about IH and what you need to show in each case.)