1 What language is accepted by the PDA

\[ M = (\{ q_0, q_1, q_2 \}, \{ a, b \}, \{ a, b, Z_0 \}, q_0, Z_0, \{ q_2 \}, \delta) , \]

with “nonempty moves” in \( \delta \) as shown below? (No need to justify your answer.)

\[
\begin{align*}
\delta(q_0, a, Z_0) &= \{(q_0, aZ_0)\} \\
\delta(q_0, b, Z_0) &= \{(q_0, bZ_0)\} \\
\delta(q_0, a, a) &= \{(q_0, aa)\} \\
\delta(q_0, a, b) &= \{(q_1, \Lambda)\} \\
\delta(q_0, b, a) &= \{(q_1, \Lambda)\} \\
\delta(q_0, b, b) &= \{(q_0, bb)\} \\
\delta(q_1, \Lambda, Z_0) &= \{(q_2, Z_0)\} \\
\delta(q_1, a, b) &= \{(q_1, \Lambda)\} \\
\delta(q_1, b, a) &= \{(q_1, \Lambda)\}
\end{align*}
\]

2 What language is accepted by the PDA

\[ M = (\{ q_0, q_1 \}, \{ a, b \}, \{ a, b, Z_0 \}, q_0, Z_0, \{ q_1 \}, \delta) , \]

with “nonempty moves” in \( \delta \) as shown below? (No need to justify your answer.)

\[
\begin{align*}
\delta(q_0, a, Z_0) &= \{(q_0, aZ_0)\} \\
\delta(q_0, b, Z_0) &= \{(q_0, bZ_0)\} \\
\delta(q_0, a, a) &= \{(q_0, a)\} \\
\delta(q_0, a, b) &= \{(q_0, b), (q_1, \Lambda)\} \\
\delta(q_0, b, a) &= \{(q_0, a), (q_1, \Lambda)\} \\
\delta(q_0, b, b) &= \{(q_0, b)\}
\end{align*}
\]

3 What language is accepted by the PDA

\[ M = (\{ q_0, q_1, q_2 \}, \{ a, b \}, \{ a, b, Z_0 \}, q_0, Z_0, \{ q_2 \}, \delta) , \]

with “nonempty moves” in \( \delta \) as shown below? (No need to justify your answer.)

\[
\begin{align*}
\delta(q_0, \Lambda, Z_0) &= \{(q_2, Z_0)\} \\
\delta(q_0, a, Z_0) &= \{(q_0, aZ_0)\} \\
\delta(q_0, a, a) &= \{(q_0, b)\} \\
\delta(q_0, a, b) &= \{(q_0, ab)\} \\
\delta(q_0, b, b) &= \{(q_1, \Lambda)\} \\
\delta(q_1, \Lambda, Z_0) &= \{(q_0, Z_0)\} \\
\delta(q_1, b, b) &= \{(q_1, \Lambda)\}
\end{align*}
\]
4 What language is accepted by the PDA

\[ M = (\{q_0, q_1\}, \{a, b\}, \{a, b, Z_0\}, q_0, Z_0, \{q_1\}, \delta) \]

where the “nonempty moves” of \( \delta \) are as follows? (No need to justify your answer.)

\[
\begin{align*}
\delta(q_0, \Lambda, Z_0) &= \{(q_1, Z_0)\} \\
\delta(q_0, a, Z_0) &= \{(q_1, aZ_0)\} \\
\delta(q_0, a, a) &= \{(q_0, aa)\} \\
\delta(q_0, a, b) &= \{(q_0, \Lambda)\} \\
\delta(q_0, b, Z_0) &= \{(q_0, bZ_0)\} \\
\delta(q_0, b, a) &= \{(q_0, \Lambda)\} \\
\delta(q_0, b, b) &= \{(q_0, bb)\} \\
\end{align*}
\]

5 What language is accepted by the PDA

\[ M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b, Z_0\}, q_0, Z_0, \{q_2\}, \delta) \]

where the “nonempty moves” of \( \delta \) are as follows? (No need to justify your answer.)

\[
\begin{align*}
\delta(q_0, \Lambda, Z_0) &= \{(q_2, Z_0)\} \\
\delta(q_0, a, Z_0) &= \{(q_0, aZ_0)\} \\
\delta(q_0, a, a) &= \{(q_0, aa)\} \\
\delta(q_0, a, b) &= \{(q_0, \Lambda)\} \\
\delta(q_0, b, Z_0) &= \{(q_0, bZ_0)\} \\
\delta(q_0, b, a) &= \{(q_1, \Lambda)\} \\
\delta(q_0, b, b) &= \{(q_0, bb)\} \\
\delta(q_1, \Lambda, Z_0) &= \{(q_0, bZ_0)\} \\
\delta(q_1, \Lambda, a) &= \{(q_0, \Lambda)\} \\
\end{align*}
\]

6 What language is accepted by the PDA

\[ M = (\{q_0, q_1, q_2\}, \{a, b\}, \{1, 2, b, Z_0\}, q_0, Z_0, \{q_2\}, \delta) \]

where the “nonempty moves” of \( \delta \) are as follows? (No need to justify your answer.)

\[
\begin{align*}
\delta(q_0, \Lambda, Z_0) &= \{(q_2, Z_0)\} \\
\delta(q_0, a, Z_0) &= \{(q_0, 1Z_0)\} \\
\delta(q_0, a, 1) &= \{(q_0, 2)\} \\
\delta(q_0, a, 2) &= \{(q_0, 12)\} \\
\delta(q_0, a, b) &= \{(q_0, \Lambda)\} \\
\delta(q_0, b, Z_0) &= \{(q_0, bbZ_0)\} \\
\delta(q_0, b, 1) &= \{(q_1, \Lambda)\} \\
\delta(q_0, b, 2) &= \{(q_0, \Lambda)\} \\
\delta(q_0, b, b) &= \{(q_0, bb)\} \\
\delta(q_1, \Lambda, Z_0) &= \{(q_0, bZ_0)\} \\
\delta(q_1, \Lambda, 2) &= \{(q_0, 1)\} \\
\end{align*}
\]
7 Use the Pumping Lemma to show that

\[ L = \{a^ib^jc^k \mid i > j > k\} \]

is not a CFL.

8 Use the Pumping Lemma to show that

\[ L = \{x \in \{a,b,c\}^* \mid n_a(x) = 2n_b(x) = 3n_c(x)\} \]

is not a CFL.

9a Use the construction from the proof of Theorem 7.2 to obtain a PDA \( M \) that accepts the language generated by the CFG with productions

\[
S \rightarrow A \mid SA \\
A \rightarrow aAb \mid B \\
B \rightarrow bBa \mid \Lambda
\]

9b Show two different accepting computations of \( M \) on input \( \Lambda \).

9c Show a computation of \( M \) that crashes on input \( \Lambda \).

9d Show two accepting computations of \( M \) for input \( abab \).

10 Find a CFG \( G \) s.t.

\[ L(G) = \{x \in \{a,b\}^* \mid n_a(x) \leq n_b(x)\} . \]

(No need to justify your answer.)