1. Find the rook polynomial for each of the following boards.

(a) 

(b) 

(c) 

2. Find the rook polynomial for a $2 \times n$ board (assuming $n \geq 2$).

3. More generally, show that the rook polynomial for an $m \times n$ board, $R_{m,n}(x)$ satisfies $R_{m,n}(x) = R_{m-1,n}(x) + nxR_{m-1,n-1}$. Use this to find the rook polynomials for $3 \times n$ and $4 \times n$ boards.

Some extra credit.

4. Find rook polynomials for each of the following shapes.

(a) $B_n$, the extension of 1b to $n$ steps. That is, $B_n$ has $4n$ squares, $n$ steps of size 4, each overlapping the step above by one square.

(b) $T_n$, the $n \times n$ staircase shape. Here, $T_4$ would be

(c) The $2n \times 2n$ version of (b), but with $2 \times 2$ blocks. The $n = 3$ case would be

5. Use rook polynomials, or just ordinary inclusion/exclusion to solve the holiday gift exchange problem my family has: Given $n$ couples ($2n$ people total) how many ways are there to assign names in a gift exchange so that you don’t get your own name, or your spouse? This is the same as permutations of $2n$ where neither 1 nor 2 are in positions 1 or 2, neither 3 nor 4 are in positions 3, 4, and so on. What is the limiting probability that a random permutation will satisfy these conditions? I will give partial credit to the case for my family: 6 couples.