Math 3298 Homework 1

The following questions are closely based on questions appearing on final exams from Calculus II from the Spring 2006 semester. Please let me know if you have serious difficulties with any of them.

1. Solve the equation \( \ln(7x - 3) = 2 \) for the exact value of \( x \).

2. Compute the derivative of \( \ln(\arctan(x) + 1) \).

3. Assume the following: \( f(-1) = 1, f'(-1) = 2 \). What does this imply about the derivative of the inverse? Draw a figure to help explain your answer.
   A. \( f^{-1'}(1) = -\frac{1}{2} \) B. \( f^{-1'}(1) = \frac{1}{2} \) C. \( f^{-1'}(-1) = \frac{1}{2} \) D. \( f^{-1'}(-1) = -\frac{1}{2} \) E. \( f^{-1'}(2) = 1 \)

4. Give an example of a series that converges absolutely.

5. Evaluate \( \lim_{x \to 0} \frac{\sin(5x)}{3x} \). Justify your answer.

6. Find the derivative of \( x^{2x} \).

7. Write the equivalent partial fractional decomposition for the following rational function:
   \[
   \frac{3x + 1}{(x - 4)(x - 3)}
   \]

8. Set up an integral with respect to \( x \) for the arclength along the parabola \( y = 3x^2 \) from the point \((0, 0)\) to \((3, 27)\). Do not evaluate the integral.

9. Find the second-order Taylor polynomial \( T_2(x) \) for the function \( f(x) = \arctan x \) expanded around \( a = 1 \). Show your work.

10. Let \( f(x, y) = x^2 + xy^4 \). Compute \( \frac{\partial f}{\partial x}(x, y) = f_x(x, y) \) and \( \frac{\partial^2 f}{\partial x \partial y}(x, y) = f_{xy}(x, y) \)

11. For the function \( f(x) = 2x + e^x \), use a linear approximation at \( x = 0 \) to estimate the value of \( f \) at \( x = -\frac{1}{2} \).

The following questions are optional but I strongly encourage you to answer them. Completing this should only take a few minutes and I greatly appreciate it.

12. Where did you take Calc I and Calc II (UMD, high school, etc.)?

13. What is your goal for your grade in this class?

14. What are some of your study techniques for math class (e.g. note-taking, flash cards, study groups, reading, etc.)?