Homework 8, due Friday March 25

(1) Write down explicit triple integrals for the mass and center of mass of the hemisphere \( x^2 + y^2 + z^2 \leq 1, \ z \geq 0 \), with density \( \rho = \sqrt{x^2 + y^2 + z^2} \). You do not have to evaluate these integrals (!).

(2) Compute the integral \( \int \int \int_R e^{x^2+y^2+z^2} \ dV \) where \( R \) is the region inside the sphere \( x^2 + y^2 + z^2 = 9 \) and within the first octant (i.e. \( x \geq 0, \ y \geq 0, \ z \geq 0 \)). A numerical answer is acceptable.

(3) Find the volume of a wedge cut from a ball of radius \( R \) by two planes which intersect on a diameter of the ball at an angle of \( \pi/6 \). (This volume is like a section of an orange.)

(4) Compute the Jacobian \( \frac{\partial(x,y)}{\partial(u,v)} \) of the transformation \( x = (u^2 - v^2)/2, \ y = (u^2 + v^2)/2 \).

(5) Use the transformation \( x = \sqrt{2}u - \sqrt{2/3}v, \ y = \sqrt{2}u + \sqrt{2/3}v \) to compute the integral \( \int \int_R (x^2 - xy + y^2)^{1/2} \ dA \) where \( R \) is the region bounded by the ellipse \( x^2 - xy + y^2 = 2 \).

(6) Match the vector field plots on the second page to the following vector fields (as usual the x-axis is the horizontal axis in these plots):

(I) \( \vec{F} = (x - 1, x + 3) \)

(II) \( \vec{F} = (y, x) \)

(III) \( \vec{F} = (\sin(x), 1) \)

(IV) \( \vec{F} = (y, 1/x) \)

(7) Compute the scalar line integral \( \int_C y \ ds \) where \( C \) is the curve \( x = y^2 \) for \( y \in [0, 3] \).

(8) Compute the scalar line integral \( \int_C x/y \ ds \) where \( C \) is the curve \( x = t^3, \ y = t^4, \ t \in [1/2, 1] \).

(9) Compute the scalar line integral \( \int_C xy^3 \ ds \) where \( C \) is the curve \( x = 3 \sin(t), \ y = 3 \cos(t), \ z = 4t \) for \( t \in [0, \pi/2] \).

(10) Is the vector line integral \( \int_C \vec{F} \cdot \ d\vec{s} \) positive, negative, or zero for the \( \vec{F} \) and \( C \) shown below? (The curve \( C \) is in blue.)
A. 

B. 

C. 

D.