4.2 The answers below are shown only using minterms, but your answer should be shown in a form of K-maps.
(a) \( f(w,x,y,z) = \sum(2,3,4,5,10,13) \)
(b) \( f(w,x,y,z) = \sum(0,1,3,4,6,7,8,9,10,14) \)
(c) given in the problem
(d) \( f(w,x,y,z) = \sum(1,2,5,6,8,10,11,12,15) \)
(e) \( f(x,y,z) = \sum(1,2,3,4,5) \)
(f) \( f(x,y,z) = \sum(1,5,6) \)

4.7 (a) \( f(x,y,z) = x + yz' \) minimal sum of products (SOP)
\( f(x,y,z) = (x + y)(x+z') \) minimal product of sums (POS)
(b) \( f(x,y,z) = x' + y + z' \) minimal SOP
\( f(x,y,z) = (x'+y+z') \) minimal POS
(c) \( f(x,y,z) = x'z' + yz \) minimal SOP
\( f(x,y,z) = (y+z')(x'+z) \) minimal POS
(d) \( f(x,y,z) = f + x'z' \) minimal SOP
\( f(x,y,z) = (w'+x')(w'+y')(x+y'+z') \) minimal POS

4.9 Remember that minimal SOP and POS are not necessarily unique.
(a) Minimal SOP
\( f(w,x,y,z) = w'x'z' + xyz + x'yz' + wx'y'z + w'xy \)
\( f(w,x,y,z) = w'x'z' + xyz + x'yz' + wx'y'z + w'yz' \)
Minimal POS
\( f(w,x,y,z) = (x'+y)(w'+y+z)(x+y'+z')(w'+x'+z)(w+x+z') \)
\( f(w,x,y,z) = (x'+y)(w'+y+z)(x+y'+z')(w'+x'+z)(w+y+z') \)
(b) Minimal SOP: \( f(w,x,y,z) = x'y' + w'x + w'z' \)
Minimal POS: \( f(w,x,y,z) = (w'+x')(w'+y')(x+y'+z') \)
(c) Minimal SOP: \( f(w,x,y,z) = x'y'z' + xyz + w'xy + w'y'z \)
Minimal POS
\( f(w,x,y,z) = (x+y')(x'+y+z)(w'+y+z')(w'+y'+z) \)
\( f(w,x,y,z) = (x+y')(x'+y+z)(w'+y+z')(w'+x'+z) \)
(d) Minimal SOP: \( f(w,x,y,z) = y'z + w'z + x'z + w'xy' \)
Minimal POS: \( f(w,x,y,z) = (x+z)(w'+z)(y'+z)(w'+x'+y') \)
(e) Minimal SOP
  \[ f(w,x,y,z) = xyz + xy'z' + wy'z' + w'x'y'z + wxy \]
  \[ f(w,x,y,z) = xyz + xy'z' + wy'z' + w'x'y'z + wxz' \]

  Minimal POS
  \[ f(w,x,y,z) = (x+y')(x'+y+z')(w+x+z)(w+y'+z)(w'+x+z') \]

(f) Minimal SOP
  \[ f(w,x,y,z) = y'z' + wz + w'y \]
  \[ f(w,x,y,z) = w'z' + wy' + yz \]

  Minimal POS
  \[ f(w,x,y,z) = (w+y+z')(w'+y'+z) \]

(g) Minimal SOP
  \[ f(w,x,y,z) = wx + y'z + wy' + xz \]

  Minimal POS
  \[ f(w,x,y,z) = (w+z)(x+y') \]

(h) Minimal SOP
  \[ f(w,x,y,z) = x'z' + xz + wy' + yz' \]
  \[ f(w,x,y,z) = x'z' + xz + wy' + xy \]

  Minimal POS
  \[ f(w,x,y,z) = (w+x+z')(x+y'+z')(w+x'+y+z) \]

(i) Minimal SOP: \[ f(w,x,y,z) = x'z + wz' \]

  Minimal POS: \[ f(w,x,y,z) = (x'+z')(w+z) \]

(j) Minimal SOP
  \[ f(w,x,y,z) = x'y' + w'y' + wxz + xyz' \]
  \[ f(w,x,y,z) = x'y' + y'z + w'xz' + wxy \]

  Minimal POS
  \[ f(w,x,y,z) = (x+y')(w+y'+z')(w'+x'+y+z) \]

4.12

(a) Minimal SOP
  \[ f(w,x,y,z) = w'x + wx'y + wz \]
  \[ f(w,x,y,z) = w'x + wx'y + y'z \]

  Minimal POS
  \[ f(w,x,y,z) = (y+z)(w+x)(w'+x'+y') \]
  \[ f(w,x,y,z) = (y+z)(w+x)(w'+x'+z) \]

(b) Minimal SOP
  \[ f(w,x,y,z) = wx'y' + w'y'z + wxy \]
\[ f(w,x,y,z) = wx'y' + w'y'z + xyz' \]

**Minimal POS**
\[ f(w,x,y,z) = (x+y')(w+z)(w'+x'+y)(y'+z') \]
\[ f(w,x,y,z) = (x+y')(w+z)(w'+x'+y)(w+y') \]

(c) **Minimal SOP**
\[ f(w,x,y,z) = x'z' + w'y' + wxz \]

**Minimal POS**
\[ f(w,x,y,z) = (x+z')(w'+x'+z)(w+x'+y') \]
\[ f(w,x,y,z) = (x+z')(x'+z')(w'+y') \]

(d) **Minimal SOP**
\[ f(w,x,y,z) = w'x'z' + w'xz' + wxy' \]
\[ f(w,x,y,z) = w'x'z' + w'xz' + xy'z' \]

**Minimal POS**
\[ f(w,x,y,z) = (x+z)(x'+z')(w'+y') \]
\[ f(w,x,y,z) = w'x'z + w'xz' + wxy' \]
\[ f(w,x,y,z) = w'x'z + w'xz' + xy'z' \]

(e) **Minimal SOP**
\[ f(w,x,y,z) = w'y' + y'z + w'x' \]
\[ f(w,x,y,z) = w'y' + y'z + x'y \]

**Minimal POS**
\[ f(w,x,y,z) = (w'+z)(x'+y') \]
\[ f(w,x,y,z) = w'y' + y'z + w'x' \]
\[ f(w,x,y,z) = w'y' + y'z + x'y \]

(f) **Minimal SOP**
\[ f(w,x,y,z) = xz' + w'y + x'y'z \]

**Minimal POS**
\[ f(w,x,y,z) = (x'+z')(x+y+z)(w'+x+y') \]
\[ f(w,x,y,z) = w'y' + y'z + w'x' \]
\[ f(w,x,y,z) = w'y' + y'z + x'y \]

(g) **Minimal SOP**
\[ f(w,x,y,z) = wx + w'x'y + wy'z' + xz' \]

**Minimal POS**
\[ f(w,x,y,z) = (w+y)(w+x'+z')(w'+x+y')(w'+x+z') \]
\[ f(w,x,y,z) = (w+y)(w+x'+z')(w'+x+y')(x+y+z') \]
\[ f(w,x,y,z) = (x+z)(x'+y+z)(w'+y'+z) \]
\[ f(w,x,y,z) = (w+x)(w+y+z)(x+y+z')(w'+y'+z) \]
\[ f(w,x,y,z) = (w+x)(w+y+z)(x+y+z')(x+y'+z) \]

(h) **Minimal SOP**
\[ f(w,x,y,z) = w'y + wz' + w'xz \]

**Minimal POS**
\[ f(w,x,y,z) = (w'+z')(w+y+z)(w+x+y) \]
\[ f(w,x,y,z) = (w'+z')(w+y+z)(x+y+z') \]
\[ f(w,x,y,z) = (w'+z')(w+y+z)(x+y'+z) \]
\[ f(w,x,y,z) = (w+x)(w+y+z)(x+y+z')(w+y'+z) \]
\[ f(w,x,y,z) = (w+x)(w+y+z)(x+y+z')(x+y'+z) \]

(i) **Minimal SOP**
\[ f(w,x,y,z) = xy + xz + wy'z' \]

**Minimal POS**
\[ f(w,x,y,z) = (w+x)(w+y+z)(x+y+z')(w'+y'+z) \]
\[ f(w,x,y,z) = (w+x)(w+y+z)(x+y+z')(x+y'+z) \]

(j) **Minimal SOP**
\[ f(w,x,y,z) = xy + wz' + w'y'z \]

**Minimal POS**

\[ f(w,x,y,z) = (w+z)(x+y')(w'+y+z') \]

### 4.14

(a) **Minimal SOP**

\[ f(v,w,x,y,z) = v'y'z + vwy + vxy' + v'wx'z \]

**Minimal POS**

\[ f(v,w,x,y,z) = (w+y')(v+z)(v'+x+y)(v+x'+y') \]

(b) **Minimal SOP**

\[ f(v,w,x,y,z) = yz + v'wy' + vw'y'z' + vwxy \]

**Minimal POS**

\[ f(v,w,x,y,z) = (v'+w'+y)(v+y'+z)(w+y'+z)(v+w+y)(v'+y+z') \]

(c) **Minimal SOP**

\[ f(v,w,x,y,z) = vwz' + w'x'z + vy'z' + v'w'xy' + v'wxy + v'xyz \]

**Minimal POS**

\[ f(v,w,x,y,z) = (v+x+z)(v+w'+y)(v'+w'+z')(w+x'+y')(v'+x'+z')(w+y'+z) \]

(d) **Minimal SOP**

\[ f(v,w,x,y,z) = w'z + y'z + wx'y + vwy' \]

**Minimal POS**

\[ f(v,w,x,y,z) = (w+z)(w'+x'+y')(v+y+z) \]

### 4.15 Minimum sums

\[ f = v'wz' + uvw' + v'w'x + uvz' \]

\[ f = v'wz' + uwv' + v'w'x + uwz' \]

**Minimum products**

\[ f = (u+v')(w'+z')(v+w+x) \]