Econ 3023 Microeconomic Analysis

Chapter 2 & 10: Budget Constraint

Instructor: Hiroki Watanabe
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Questions & Objectives

Definitions

Trinity

Some Useful Assumptions

Comparative Statics

Applications

Now We Know
Questions

- How does price increase in gasoline affect your decision on commuting?
- How is interest rate related to housing?

Objectives

- Consumer theory predicts how consumers choose their combination of goods under constraints.
  - **Consumers choose**
    - I’d like to have 5000 cheesecakes and 8000 cups of tea.
    - I’d rather have 3 cheesecakes and 4 cups of tea than 8 cheesecakes and no tea.
  - Two aspects to consider:
    - Do not mix them up.
  - Do not mix them up. part alone doesn’t predict the consumer’s choice.

Consumer Theory Overview

- What do consumers face?
  - Chapter 2 & 10
- What do consumers want?
  - Chapter 3 & 4
- How do consumers resolve conflict above?
  - Chapter 5
### Budget Constraints

**Definition 2.1 (Consumption Bundle)**

A consumption bundle contains \( x_1 \) units of commodity 1, \( x_2 \) units of commodity 2. It is denoted by \( x = (x_1, x_2) \).

- \( x = (2, 5) \) denotes a bundle of 2 cheesecakes and 5 teas.
- Commodity prices are denoted \( p = (p_1, p_2) \).
- \( p = (3, 1) \) means cheesecakes and tea are sold for \$3 and \$1 each.
- Expenditure on \( x = (2, 5) \) is \( p_1 x_1 + p_2 x_2 = \) ______.$

**Definition 2.2 (Budget Constraints & Budget Lines)**

1. A collection of bundles that are affordable forms the consumer’s budget constraint. A bundle \( x \) satisfies your budget constraint if

\[
p_1 x_1 + p_2 x_2 \leq m,
\]

where \( m \) denotes your income.

2. A budget line is a collection of bundles \( x \) that are just affordable, i.e.,

\[p_1 x_1 + p_2 x_2 = m.\]
Graphical Representation

Suppose \((p_c, p_T) = (3, 2)\) and your income is $12.

- The budget line is \(3x_c + 2x_T = 12\), or equivalently,
  \[ x_T = -\frac{3}{2}x_c + 6. \]

We can represent the budget line on the x-y plane.

1. Recall \(x_T = -\frac{3}{2}x_c + 6\) on the graph.

2. Continuity assumption: We assume that commodities are sold in any measure, i.e., \(x_1\) is not necessarily a whole number. Tea is sold by ounces rather than by cups.

Fact 2.3 (Intercepts and Budget Constraint)

- The **y-intercept** denotes the quantity of good 2 when Liz doesn’t consume good 1.\[\frac{6}{3/2}\].
- The **x-intercept** denotes the quantity of good 1 when Liz doesn’t consume good 2.\[\frac{6}{3/2}\].
Questions & Objectives

Definitions

Trinity
- Slope of the Budget Line
- Relative Price
- Opportunity Cost

Some Useful Assumptions

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Slope of the Budget Line

Question 3.1 (Slope of the Budget Line)
What does the slope of the budget line imply?

Answer 3.2 (Trinity on the Budget Side)
The following are the same:
- The slope of the budget constraint.
- Relative Price.
- Opportunity Cost.
Relative Price

1 A slice of cheesecake buys \( \frac{p_C}{p_T} \) cups of tea.
2 If you sell one slice of cheesecake, then you’ll receive \( p_C \) dollars. With \( p_C \) dollars, you can buy \( \frac{p_C}{p_T} \) cups of tea at the market.
3 The slope tells you the market rate of exchange between \( \chi_C \) and \( \chi_T \), a.k.a. relative price of tea in terms of cheesecakes.  

Footnote: Note that the relative price is predetermined in the market and your preferences have no bearing on it.

Opportunity Cost

Definition 3.3 (Opportunity Cost)

is the best alternative that Liz forgoes or gives up, when she makes a choice or decision.

1 When Liz buys one more slice of cheesecake, Liz has to give up consuming \( \frac{p_T}{p_C} \) units of tea.  

Footnote: Once again, we do not care why Liz would trade cheesecakes and tea here. It is simply a market matter but not Liz’s preference matter.

Opportunity Cost

To sum up, the slope of the budget line represents

- the relative price, and
- the opportunity cost of buying (consuming) one unit of \( \chi_C \) (measured in terms of tea).

Note: we do not care who the consumer is at this point. Prices are given in advance.
### Questions & Objectives

### Definitions

### Trinity

### Some Useful Assumptions
- **Numéraire**
- **Composite Goods**

### Comparative Statics

### Applications

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#### Numéraire

You **do not need to know individual prices** to find budget constraints.
- You only need to know:
  1. Liz's income
  2. Relative price (i.e., market rate of exchange) of commodities.

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#### Definition 4.1 (Numéraire Good)

A commodity whose price is normalized to one.
- **Numéraire** means:
  - Any commodity can be chosen as a numéraire without changing the budget set or the budget constraint.
  - We can **reduce** the number of parameters in this manner.
When we buy a house, we do not think of cheesecakes that we have to give up.

Rather, we loosely think of "other stuff" that we could buy if we did not buy a house.

Economists have a nice hypothetical commodity called

Definition 4.2 (Composite Good)

is a basket of commodities consisting of goods other than a good in question (a house for example).

A composite good \( x_2 \) is a basket of one cheesecake, two cups of tea, a pair of shoes, your phone bill, etc.

A price of composite good \( p_2 \) is a cost of purchasing one basket.

Budget constraint

\[
p_1x_1 + p_2x_2 + p_3x_3 + \cdots + p_Nx_N \leq m.
\]

simplifies to

\[
p_1x_1 + p_2x_2 \leq m
\]

expenditure on a house exp. on other goods
Questions & Objectives
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- Change in Income
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Change in Income

Question 5.1 (Change in Parameters and Budget Line)
How does the budget line change when prices and income change?
- Stay put
- Parallel shift
- Rotation (pivotting)
- Combination of parallel shift and rotation

Suppose the income increases from 1 to 100:

\[ p_1 x_1 + p_2 x_2 = 1 \]
\[ p_1 x_1 + p_2 x_2 = 100. \]
Change in Income

- The intercept changes because ...
- The slope remains constant because ...

\[ x_2 = \frac{-p_1}{p_2} x_1 + \frac{1}{p_2} \rightarrow x_2 = \frac{-p_1}{p_2} x_1 + \frac{100}{p_2}. \]

- A change in income results in a parallel shift.

Change in Prices

- Suppose \( p_1 \) increases from 2 to 100:

\[ 2x_1 + p_2 x_2 = m \]
\[ 100x_1 + p_2 x_2 = m. \]

- The slope changes because ...
- The horizontal intercept changes because ...
- The vertical intercept remains constant (why?)
- A change in price results in rotation.

\[ x_2 = \frac{-2}{p_2} x_1 + \frac{m}{p_2} \rightarrow x_2 = \frac{-100}{p_2} x_1 + \frac{m}{p_2}. \]
Exercise 5.2 (Change in Price)

Liz spends $12 on cheesecakes \((x_C)\) and tea \((x_T)\). The price is given by \((p_C, p_T) = (3, 2)\).

1. Draw Liz's budget line.
2. What happens to the budget line if \(p_T\) drops to 1? In particular, consider the change in
   - maximum amount of cheesecakes she can purchase
   - maximum amount of tea she can purchase
   - her opportunity cost of a cheesecake in terms of tea
3. Can you say the rotation of budget line leave Liz happier than before? If not, what additional information do you need?
While the reduction in tea’s price relaxes Liz’s budget constraint, it does not necessarily increase her happiness.

She may not

We also need to know Liz’s preferences as well as her budget constraint to understand her consumption behavior (to be discussed in next lecture).

The budget constraint can represent a wide range of commodities.

Examples include budget constraints over time and savings.
Example 6.1 (Intertemporal Budget Constraint)

- Liz’s monthly income is $4,000 this month and $0 in July.
- She will take a month off to spend July in Cancun for her summer vacation.
- She is planning to save some of her income this month for her trip next month.
- Monthly interest rate is 10%.

Question 6.2 (Representing Intertemporal Budget)

How can we represent her bundle? What would be $x_C$ and $x_T$ in this case?

- Note we are not concerned with cheesecakes this month or tea next month.
- Rather, we are concerned with what Liz can spend this month and next month.

Solution: Represent this month’s consumption by a composite good $x_6$ and next month $x_7$.

- June’s basket ($x_6$) might include meatball subs, pens, gasoline, and
- July’s basket ($x_7$) might include mojitos, sunscreens and sombreros,
- which we do not care about. We’d like to see the relationship betw. affordability and savings.
Intertemporal Budget Constraints

- Let \( p_6 = p_7 = 1 \) for simplicity (i.e., both commodities are numéraire).

Example 6.3 (Affordable Bundles)

Take \( x_6 \) on x-axis and \( x_7 \) on y-axis. Where do the following situations appear on your graph?

- The bundle \((x_6, x_7)\) when Liz doesn’t save at all.
- The bundle when Liz saves all of her income for next month.
- The bundle when Liz uses $1,500 this month and save the rest for next month.
- The bundle when Liz uses $2,000 this month and save the rest for next month.

It looks like the bundles are on the same line. They are:

\[
x_7 = (1 + .1) (2000 - x_6),
\]

or equivalently,

\[
x_7 = -1.1 x_6 + 4400. \tag{1}
\]

The budget line (1) is called intertemporal budget constraint in particular.
Recall the trinity.

The opportunity of $x_7$ is 1.1, i.e., if Liz gives up one unit of $x_6$, then she will get 1.1 units of $x_7$ in July.

In the previous example, the value of a basket does not change over time.

Reason: the price does not change over time.

What happens if the prices change from month to month?

In particular, how is Liz’s purchasing power affected by the change in prices?
\textbf{Fisher Equation}

- To keep things simple, suppose there is one commodity in the world: tea leaves.
- Consider two periods 1 and 2.
- We have a bundle \((x_1, x_2)\) and price \((p_1, p_2)\).

\textbf{Definition 6.4 (Inflation Rate and Nominal Interest Rate)}

\textbf{Inflation rate} \(\pi\): measures the growth in prices:
\[ 1 + \pi = \frac{p_2}{p_1}. \] (2)

\textbf{Nominal interest rate} \(i\): if you save a dollar in period 1, you’ll receive \(1+i\) dollars in period 2. Likewise, to receive a dollar in period 2, you need to save \(\frac{1}{1+i}\) dollars in period 1.

If you save \(p_1\) dollars, you’ll get \((1+i)p_1\) dollars in period 2.
- That doesn’t imply you can simply purchase \(1+i\) tea leaves in the next period.

Consider the following to track down the change in Liz’s purchasing power:
- To buy 1 \textit{basket} in period 1, Liz needs \(p_1 s_1\).
- i.e., the purchasing power of \(p_1 s_1\) is 1 basket.
- \(p_1 s_1\) does not guarantee the same purchasing power in period 2.
- \(p_1 s_1\) translates to \(p_1 (1+i)s_2\) in period 2.
- How many tea leaves does \(p_1 (1+i)s_2\) buy?
\[ \frac{1}{p_2 s_2 / \text{basket}} = \frac{p_1 (1+i)}{p_2} \text{ baskets.} \]
- The ability to buy 1 basket in period 1 translates to the ability to buy \(\frac{p_1 (1+i)}{p_2}\) basket in period 2.
Definition 6.5 (Real Interest Rate)

Real interest rate \( r \) measures the growth in purchasing power:

\[
1 + r = \frac{p_2(1 + i)}{p_1}.
\]

Recall (2): \( 1 + \pi = \frac{p_2}{p_1} \). Then we can write (3) as

\[
1 + r = \frac{1 + i}{1 + \pi},
\]

or in other words,

\[
\text{(change in purchasing power)} \times \text{(inflation rate)} = \text{(nominal interest rate)}.
\]

The relationship above is called **Fisher equation**.

Exercise 6.6 (Fisher Equation)

Compute the real interest rate in Liz’s Cancun example. Recall

- nominal interest rate is \( i = .1 \).
- \( p_6 = p_7 = 1 \).
- Budget constraint
- Trinity
- Numéraire
- Composite goods
- Comparative statics

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