# Econ 3023 Microeconomic Analysis

## Chapter 15B: Elasticity

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<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>Elasticity</td>
<td>Optimal Pricing &amp; Elasticity</td>
<td>Summary</td>
</tr>
</tbody>
</table>

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How does the change in price affect our consumption choice?

Slutsky decomposition (Ch. 14) and its precursor Ch. 6

One problem: unit sensitivity
  \( \Delta x \) changes depending on the unit of measure.

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Question 1.1 (Hawaii Hotel Tax)

Take a listen to [Marketplace Clip].

Some state lawmakers want to boost one of the taxes on hotel rooms – from around 7 percent to around 12 percent.

But the proposed increase could keep corporate visitors away.

Does a proposed tax hike lead to larger tax revenue?

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Two forces at play:

- State lawmakers: Increase in revenue because they can collect larger tax from each tourist.
- Bill Connors: Reduction in revenue because increased tax reduces the number of tourists.

Q: Which force surpasses the other?
Introduction

Elasticity
  • Percent Change
  • Price Elasticity of Demand

Optimal Pricing & Elasticity

Summary

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Definition 2.1 (Percent Change)

**Percent change** in a variable $x$ is denoted by

$$\frac{x_{\text{new}} - x_{\text{old}}}{x_{\text{old}}} \times 100 = \frac{\Delta x}{x_{\text{old}}} \times 100.$$ 

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Suppose 1.5$\$/€.

<table>
<thead>
<tr>
<th>Price</th>
<th>$\Delta p$</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>€50</td>
<td>$€10$</td>
<td>20%</td>
</tr>
<tr>
<td>$75</td>
<td>$90$</td>
<td>20%</td>
</tr>
</tbody>
</table>

% change is a unit-independent measure.
Price Elasticity of Demand

Definition 2.2 (Price Elasticity of Demand)
Price elasticity of demand measures the responsiveness of demand against the change in price, defined by
\[
\epsilon(x) := \left| \frac{\% \text{ change in } x}{\% \text{ change in } p} \right| = \left| \frac{\Delta x / x}{\Delta p / p} \right|.
\]

\*\(|a|\) means \(a\) if \(a\) is positive and \(-a\) if \(a\) is negative.

- Usually, \(\epsilon(x) = -\Delta x / x \Delta p / p\).
- Regardless of the sign of \(a\), \(|a|\) is always positive.

The slope \((\Delta p / \Delta x)\) is one way to measure the responsiveness, but then the exact same demand curve is steeper in US than in Europe. Quantity demanded increases by \(\epsilon(x)\%\) against 1% increase in price.

We can rearrange
\[
\epsilon(x) = \left| \frac{\Delta x / x}{\Delta D(x) / D(x)} \right| = \left| \frac{x \Delta D(x)}{\Delta x D(x)} \right| = \left| \frac{D(x) / x}{\text{slope of the inverse demand curve}} \right|.
\]

You can also write \(\epsilon(x) = -\log x\).
Exercise 2.3 (Elasticity for a Linear Demand Function)
Suppose the inverse demand function is given by $D(x) = \frac{-1}{2}x + 10$. The slope ($\frac{\Delta D(x)}{\Delta x}$) is $-\frac{1}{2}$. Find the elasticity at $x = 5, 10, 15$.

- $e(x) = \frac{\frac{\Delta D(x)}{\Delta x}}{\frac{D(x)}{x}} = \frac{x-20}{x}$.
- $e(5) = 3$.
- $e(10) = 1$.
- $e(15) = \frac{1}{3}$.

Even when the slope is constant everywhere, elasticity varies.
**Price Elasticity of Demand**

**Definition 2.4 (Elastic & Inelastic Demand)**

- Demand is **elastic** at \( x \) if \( \epsilon(x) > 1 \).
- Demand is **unit elastic** at \( x \) if \( \epsilon(x) = 1 \).
- Demand is **inelastic** at \( x \) if \( \epsilon(x) < 1 \).

---

**Example 3.1 (Revenue & Elasticity)**

Jack’s total revenue is

\[
TR(x) = \text{price} \times \text{quantity} = D(x) \times x,
\]

where \( D(x) = \frac{1}{2}x + 10 \). Does it increase Jack’s revenue to sell one more cheesecake (by reducing price accordingly)?

- If additional increase in sales brings in extra revenue \( \frac{\Delta R(x)}{x} > 0 \), then yes.
Does additional sales bring in money?

\[
\frac{\Delta TR(x)}{\Delta x} = \frac{\Delta[D(x) \cdot x]}{\Delta x} = \frac{\Delta D(x)}{\Delta x} \cdot x + \frac{D(x)}{\Delta x} \cdot \Delta x
\]

- price differential
- existing sales
- revenue from an additional cheesecake

Q': In what range of \( x \) is it a good idea to increase sales of \( x \) by reducing \( p \)?

Increasing sales by reducing the price will bring in more revenue if current sales (\( x \)) falls into the elastic region. ²

- 1% price reduction leads to more than 1% sales increase.
- price effect > quantity effect.

²See Appendix for proof.
- If current sales ($x$) falls into the inelastic region, increasing sales by reducing the price will bring in less revenue.
- 1% price reduction leads to less than 1% sales decrease.
- price effect < quantity effect

If current sales ($x$) falls into the unit-elastic region, increasing sales by reducing the price will not change revenue.
- 1% price reduction leads to 1% sales decrease.
- price effect = quantity effect
- Jack can't do any better, i.e., this is where he maximizes the revenue.
Chapter 24 preview:
- We’ve just found the level $x_C$ where Jack maximizes total revenue for Example 3.1.
- The cost was out of equation.
- If Jack takes the cost into account, Jack never operates in the inelastic region.
Measuring the responsiveness of demand against price.
- When to raise price.

\[
\frac{\Delta R(x)}{\Delta x} > 0
\]
\[
\frac{\Delta D(x)}{\Delta x} x + D(x) > 0
\]
\[
1 + \frac{\Delta D(x)}{\Delta x} < 0
\]
\[
1 + (-\varepsilon(x)) < 0
\]
\[
1 < \varepsilon(x).
\]

Additional revenue by selling one more slice > 0

\[
D(x), \text{see inverse demand}
\]
\[
demand, 10
\]
\[
elastic, 16
\]
\[
\varepsilon(x), \text{see price elasticity of demand}
\]
\[
inelastic, 16
\]
\[
inverse \text{ demand, 12, 13}
\]
\[
\text{percent change, 8}
\]
\[
\text{price effect, 6}
\]
\[
\text{price elasticity of demand, 10}
\]
\[
\text{quantity effect, 6}
\]
\[
\text{Slutsky decomposition, 4}
\]
\[
\text{total revenue, 18}
\]
\[
TR(x), \text{see total revenue}
\]
\[
\text{unit elastic, 16}
\]
\[
\text{unit sensitivity, 4}
\]