Econ 3023 Microeconomic Analysis

Problem Set #2

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Spring 2013

- Due Thursday, February 28th at 11 AM in class.
- Be legible and please print.
- Label your graph.
- Your target audience is not me but your peers. A successful answer is the one that convinces your peers that your answer is correct without further explanation. If they don’t buy your argument, your answer is incomplete even when the end result is correct. Unless otherwise stated, your job is not only to leave the answer but explain why your answer is correct.
- Show your intermediate work whenever possible. This will help with partial credits in case you did not reach the correct answer.
- 1 randomly chosen question out of 5 will be graded. Make sure you review them all with the suggested solution to be posted though. You might meet any one of them again in the exam.
- The ones marked [Optional] do not count towards the score. Math freaks or economics aficionados, go ahead. Otherwise, you can skip them and review the suggested solution later.
- Some questions are easy and some are challenging (otherwise you will get bored). I marked each question as follows for your convenience:

- You can solve this blindfold. This is meant to test your basic understanding of the material.
- You might want to take off your blindfold.
- This one calls for a light bulb moment. Think it through till your light comes on.

1 UMP and Demand

Pick one:

A. 🌟🌟🌟 Using the term "relative price" and "marginal rate of substitution", outline the procedure to find the optimal bundle for standard utility.

B. 🌟🌟🌟 For non-extreme cases, explain why relative price cannot be either higher or lower than the marginal willingness to pay at the optimal bundle.

2 Utility Maximization Problem

Jerome has $m$ to spend on $x_c$ slices of cheesecakes and $x_T$ cups of tea. His preferences are represented by a utility function

$$u(x_c, x_T) = x_c x_T,$$

with marginal rate of substitution at $(x_c, x_T)$ given by $MRS(x_c, x_T) = \frac{-x_T}{x_c}$. Price of cheesecake and tea is $p = (p_c, p_T) = (2, 1)$.

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A. Find the value of his MRS at \((x^C, x^T) = (5, 10), (10, 20), (15, 30)\). What these numbers tell about the shape of indifference curves at these points?

B. Find the value of his MRS at \((x^C, x^T) = (5, 20), (10, 20), (15, 20)\). What is the rationale behind these numbers? (Tip: Recall what MRS measures. Think how much Jerome is willing to give up to get a slice at each combination).

C. What is the relative price of a slice of cheesecake measured in cups of tea? Unlike MRS’s above, the value of the relative price does not depend on \((x^C, x^T)\). Explain, in words, why MRS does and relative price does not.

D. Write Jerome’s utility maximization problem.

E. Write the tangency condition.

F. Assuming \(m = 10\), find the solution \((x^*_C, x^*_T)\) to item D.

G. Being a bit skeptical of his solution in item F, he decided to compute his utility level at \((x^*_C, x^*_T)\) and compare it to the utility level he gets from \((x^C, x^T) = (10, 0)\) and \((8, 8)\). One of the two bundles he picked will make a valid comparison to \((x^*_C, x^*_T)\) and will confirm that the solution in item F is actually the solution. The other one is irrelevant and does not help to confirm item F. Explain to him why.

H. Assume \(m = 20\) and \(m = 30\) and find the solution to item D. Is a cheesecake normal to him?

3 UMP and Demand

Andres has $m (will be either \(m = 30\) or \(60\)) to spend on \(x^C\) slices of cheesecake and \(x^T\) cups of tea. Tea is one dollar a cup and price of cheesecake is yet to be known. Let \(p_C\) denote the price. His preferences for cheesecakes and tea are represented by

\[u(x^C, x^T) = x^C x^T\]

and his marginal rate of substitution at \((x^C, x^T)\) is \(MRS(x^C, x^T) = \frac{-2x^T}{x^C}\).

A. What is the relative price of cheesecake measured in cups of tea?

B. Write his utility maximization problem.

C. Write the tangency condition.

D. Find his Marshallian demand for cheesecakes \(\varphi^C(p_C, m)\). (Tip: His Marshallian demand for tea is \(\varphi^T(p_C, m) = \frac{m}{3}\)).

E. Assume \(m = 30\). With \(x^C\) on the horizontal axis and \(p_C\) on the vertical axis, sketch the demand curve. (Tip: It has a shape of hyperbola. If you forgot what it is, you can plot the quantity demanded at \(p_C = 1, 2, 4\) and connect the dots).

F. What is his marginal willingness to pay for \(10\)th slice according to the Marshallian in item E?

G. Assume \(m = 60\) and sketch the demand curve. How does it compare to the demand in item E? Explain.

4 Slutsky Decomposition

Concerned about Matthew’s excessive consumption of cheesecake, Jay has decided to analyze his consumption behavior with a number of experiments. Matthew consumes cheesecakes \((x^C)\) and tea \((x^T)\) with his allowance of \(m = 20\). Tea is a numéraire and the price of cheesecake was originally \(p^C_0 = 2\) and now it is \(p^C_1 = 1\).

A. Jay has observed that Matthew’s original and final optimal bundles are

\[x^0 = \varphi(p^0_C, p^0_T, m) = \left(\begin{array}{c} 5 \\ 10 \end{array}\right), \quad x^f = \varphi(p^f_C, p^f_T, m) = \left(\begin{array}{c} 10 \\ 10 \end{array}\right),\]

where \(\varphi(p_C, p_T, m)\) denotes Matthew’s Marshallian demand. What is his total effect \(\Delta x\)?
B. ▶▶▶ Does cross price effect on \( x_T \) exist? Explain.

text

C. ▶▶▶ To isolate Matthew’s substitution effect, how much money should Jay take away from Matthew’s budget? Explain.

text

D. ▶▶▶ After a little battle, Jay did manage to take some cash off Matthew’s wallet. With the adjusted income Jay found in item C, Matthew picked

\[
x^T = \varphi(p_c^e, p_T, m^T) = \begin{pmatrix} 10 \\ 5 \end{pmatrix}.
\]

text

Compute income effect \( \Delta x^I \) and substitution effect \( \Delta x^S \).

text

E. ▶▶▶ From this experiment, can Jay say that Matthew’s cheesecake consumption is normal or income-inferior? Explain.

F. ▶▶▶ Are cheesecakes and tea substitutes or complements to Matthew? Explain.

5  The Principle and UMP

A. ▶▶▶ Read Landsburg Chapter 4: The Indifference Principle. The indifference principle is Principle 2.1 in Prologue put in a different way. Pick any example story from the chapter and discuss how it is related to Principle 2.1 (i.e., what happens when the situation is in equilibrium).

B. ▶▶▶ [Optional] We used (probably without knowing) Principle 2.1 when solving utility maximization problem. Reexamine the tangency condition in light of the principle and discuss how the principle was incorporated in finding the solution. (Tip: Recall that at the peak of the mountain, you cannot go either up or down. The same thing happens when you choose the right amount of cheesecakes and tea).