Econ 3023 Microeconomic Analysis

Chapter 8: Slutsky Decomposition

Instructor: Hiroki Watanabe
Spring 2013

Introduction

Decomposing Effects

Giffen Is Income-Inferior

Examples

Hicks

Now We Know
Chpt 6: Categories and definitions.
Chpt 8: Price change → demand.
Chpt 14: Price change → welfare.

Discussion 1.1 (Orange vs Tomato)

- Listen to [Link to NPR News Clip].

Orange juice is not a required staple for people to live. So they will turn to other things and that will bring down prices eventually.

- When the price of OJ goes up, do people
  1. increase consumption of tomato juice to substitute away from OJ, or
  2. can they possibly consume less tomato juice?

Exogenous Variable $\phi^c(p, m) \uparrow \phi^c(p, m) \downarrow$

$m \uparrow$ normal income inferior
$p_1 \uparrow$ Giffen LOD
$p_2 \downarrow$ substitute complement
Overview

- $\Delta x_C$ denotes change in $x_C$.
- If $x_C = 5$ and then $x_C$ becomes 2 after the price change (or income change), $\Delta x_C = -3$.
- $\Delta x$ denotes change in $x_T$.
- If $x_T = 3$ and then $x_T$ becomes 8 after the price change (or income change), $\Delta x = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$.

Background

- Consider $(p_C^0, p_T) = (100, 1)$ and $(p_C^1, p_T) = (1, 1)$.
- $\Delta x_C$ has two components:
  - Income effect, $x_C$ Å: some money left after purchasing $x_C^0$.
  - Substitution effect, $x_C$ Å: Liz has to give up only one slice for a cup of tea.
Background

Definition 1.2 (Income & Substitution Effect)

1. **Income effect** is a change in demand $\Delta x^I$ due to having more purchasing power.
2. **Substitution effect** is a change in demand $\Delta x^S$ due to change in relative price.
3. **Total effect** $\Delta x$ is the sum of the above:

$$\Delta x := \Delta x^I + \Delta x^S.$$
Introduction

2 Decomposing Effects
- Eliminating Income Effect
- Slutsky Decomposition
- Example 1: Cobb-Douglas

3 Giffen Is Income-Inferior

4 Examples

5 Hicks

6 Now We Know

Eliminating Income Effect

Goal: Eliminate the income effect (change in purchasing power) and isolate the substitution effect.

Idea: You can compensate the income to suppress the change in purchasing power.

Then let Liz solve her UMP with compensated income.
Eliminating Income Effect

\[ \text{Tea} \times T \quad (\text{cups}) \]

\[ \text{Cheese} \times C \quad (\text{slices}) \]

BL O
BL T
BL F

\[ \begin{array}{cccccccc}
0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 \\
0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 \\
\end{array} \]
Eliminating Income Effect

- We’ll take away \( m - m^T \) dollars from \( m \).
- Liz: I don’t feel any change in my purchasing power. With income adjustment, I still use up my income to purchase \( x^C \) regardless of the price.
- Or equivalently, change \((p_0^C, p_T, m) \rightarrow (p_0^C, p_T, m^T)\) reserves Liz’s purchasing power since she can afford \( x^C \) in both environments.
- Setting income at \( m^T \) suppresses the income effect and isolates the substitution effect.

Problem 2.1 (UMP):

\[
\max x_C, x_T \quad \text{s.t.} \quad p_0^C x_C + p_T x_T = m \\
\text{with the solution } x_C = \begin{bmatrix} \phi^C(p_0^C, p_T, m) \\ \phi^T(p_0^C, p_T, m) \end{bmatrix}.
\]

\[\downarrow\] pure substitution effect \( \Delta x^S \)

Problem 2.2 (UMP):

\[
\max x_C, x_T \quad \text{s.t.} \quad p_0^C x_C + p_T x_T = m^T \\
\text{with the solution } x_T = \begin{bmatrix} \phi^C(p_0^C, p_T, m^T) \\ \phi^T(p_0^C, p_T, m^T) \end{bmatrix}.
\]

\[\downarrow\] pure income effect \( \Delta x^I \)

Problem 2.3 (UMP):

\[
\max x_C, x_T \quad \text{s.t.} \quad p_0^C x_C + p_T x_T = m \\
\text{with the solution } x_T = \begin{bmatrix} \phi^C(p_0^C, p_T, m) \\ \phi^T(p_0^C, p_T, m) \end{bmatrix}
\]
Slutsky Decomposition

- Substitution effect $\Delta x^S$ is given by
  \[ \Delta x^S = \phi(p_C^*, p_T, m^*) - \phi(p_C^0, p_T, m). \]
- Income effect $\Delta x^I$ is given by
  \[ \Delta x^I = \phi(p_C^*, p_T, m) - \phi(p_C^0, p_T, m^*). \]
- Total effect $\Delta x$ is given by
  \[ \Delta x = \phi(p_C^*, p_T, m) - \phi(p_C^0, p_T, m) = \Delta x^I + \Delta x^S. \]

**Definition 2.4 (Slutsky Decomposition)**

Slutsky decomposition splits the total effect into two parts:

\[ \Delta x = \Delta x^I + \Delta x^S. \]

**Example 1: Cobb-Douglas**

**Example 2.5 (Cobb-Douglas Utility Function)**

Suppose Liz consumes cheesecakes $x_C$ and tea $x_T$. Initial price of $p_T^2 = 2$ was slashed in half to $p_T^2 = 1$. $p_T = 1$ and $m = 16$ throughout. Her preference is represented by

\[ u(x_C, x_T) = x_C x_T, \]

whose MRS at $(x_C, x_T)$ is $-x_T / x_C$. What are $\Delta x^S$ and $\Delta x^I$?
Example 1: Cobb-Douglas

Steps:
1. UMP\(O\): Find \(x^O\).
2. Find \(m^T := p^T x^O + p^T x^T\).
3. UMP\(T\): Find \(x^T\).
4. UMP\(F\): Find \(x^F\).
5. Compute \(\Delta x := \Delta x^d + \Delta x^S\).

\[
\max_{x_C, x_T} (x_C x_T) \text{ s.t. } p^O x_C + x_T = m.
\]

- MRS at \((x_C, x_T)\) is \(\frac{-x_T}{x_C}\).
- \(x^O = (4, 8)\).
Example 1: Cobb-Douglas

1. Find $m^T$.
   - $x^D = (4, 8)$.
   - $m^T := p^D C + 8 = 12$.

2. UMP:
   \[
   \max u(x_C, x_T) = x_C x_T \quad \text{s.t.} \quad p^F C + x_T = m^T.
   \]
   - $x^F = (6, 6)$.
   - $\Delta x^S = x^T - x^D = \begin{pmatrix} 6 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$.

3. Slutsky decomposition:
   \[
   \Delta x = \Delta x^F + \Delta x^S = \begin{pmatrix} 2 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}.
   \]
Proposition 2.6 (Cobb-Douglas Utility Function and Cross-Price Effect)

Cross-price effect is ______ for Cobb-Douglas utility function.

Proof.

Income effect and substitution effects cancel each other out (see above).
Fact 3.1 (Sign of Substitution Effect)

Substitution effect $\Delta x_C$ against $p_C$ is positive if preferences are convex.

- If purchasing power remains the same, decrease in $p_C$ results in increase in $x_C$.  

\[ ^1 \text{Note} \]
- LOD says $p_C$ implies $x_C$.
- The fact above says $p_C$ implies $x_C$ if the purchasing power remains the same.

Normal Good

- For a normal good with convexity,
Suppose $p_C \downarrow$.

Slutsky decomposition:

$$\Delta x_C = \Delta x^C_C + \Delta x^S_C .$$

+ by normality + by Fact 3.1.

Conclude: $p_C \downarrow \Rightarrow \Delta x_C \uparrow$.

Conclude: a normal good satisfies LOD.

For an income-inferior good with convexity,
In case of extreme income-inferiority, income effect may be larger in magnitude than the substitution effect, causing \( x_C \downarrow \) even with \( p_C \downarrow \).

Conclude: an income-inferior cheesecake may be Giffen.

Recall the definition: Giffen good: \( p_C \downarrow \Rightarrow x_C \downarrow \).

Conversely, if a cheesecake is Giffen,

\[
\Delta x_C = \Delta x_C^s + \Delta x_C^s.
\]

\( \Delta x_C^s \) has to be negative.

Conclude: Giffen cheesecake has to be income-inferior.
Proposition 3.2 (Giffen Is Income Inferior)

$m$-wise normal income-inferior

$p_c$-wise LOD Giffen

Proof.
See above.

Remark
There is no such thing as a normal Giffen cheesecake.
Example 2: Perfect Complements

- Rotation does not change $x^*$: $\Delta x^S = x^T - x^O = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.
- $\Delta x = \Delta x^f + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Example 3: Perfect Substitutes
Example 3: Perfect Substitutes

- $x^T$ is already $x^r$.
- $\Delta x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \Delta x^\pi$ (all the change is due to substitution effect).

Example 4: Quasi-Linear Preferences

- For quasi-linear utility function $f(x_C, x_T) = x_C + v(x_T)$, the composite good picks up all the income effect.
- Indifference curves are parallel to each other.
- So, if you parallel shift the budget line, $x_C$ grows in direct proportion, whereas $x_T$ stays the same.
- ... why?
Example 4: Quasi-Linear Preferences

\[ MRS = \text{constant at any given } x_T: MRS(1, 2) = MRS(30, 2) = MRS(32170984872109874, 2). \]

\[ \text{i.e., one basket is worth the same amount of tea regardless of the number of baskets.} \]

\[ \text{Baskets } \approx \text{ Liz's income (less expenditure on tea)} \]

\[ \text{She won't get saturated by cash regardless of the amount of cash.} \]

Parallel shifts cannot change \( x_T \).

\[ \text{She is willing to give up the same amount of tea (MRS cups) no matter how rich or poor she is.} \]

\[ \text{Therefore, income effect for tea is } \text{absent} \text{ for quasi-linear preferences.} \]

\[ \Delta x_T = 0 + \Delta x_T^5 \]

\(^2\)Compare this, for example, to Cobb-Douglas. MRS grows smaller as cheesecake consumption.
An alternative way to compensate (adjust) income level for Slutsky decomposition.
Slutsky: adjust $m$ to guarantee that $x^0$ is available under $p^c$.
Hicks: adjust $m$ in the way that guarantees $u(x^0)$ under $p^c$.
When $\Delta p_C$ is small, the Slutsky substitution effect is almost the same as the Hicksian substitution effect.

Comes in handy because it does not depend on the value of utility level. We’ll use this in Chapter 14.
- Decomposing total change in quantity demanded.
- Property of Giffen goods.
- Alternative compensation.