1. Suppose that you buy a share of stock, then sell it 30 days later. If the probability of making a profit of $250, $150, $0, or $150 are 0.22, 0.36, 0.28 and 0.14, respectively. What is the expected gain? What is the variance? Let $X$ be the profit random variable.

<table>
<thead>
<tr>
<th>$X$</th>
<th>250</th>
<th>150</th>
<th>0</th>
<th>-150</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(X)$</td>
<td>0.22</td>
<td>0.36</td>
<td>0.28</td>
<td>0.14</td>
</tr>
</tbody>
</table>

$E(X) = (250)(0.22) + (150)(0.36) + (0)(0.28) + (-150)(0.14) = 55 + 33 = 88$

$\text{Var}(X) = E(X^2) - (E(X))^2 = 62500(0.22) + 22500(0.36) + 22500(0.14) - 88^2$

$= 13750 + 11250 = 25000 - 7744 = 17256$

2. In a gambling game, a woman is paid $3 if she draws a jack or a queen and $5 if she draws a king or an ace from an ordinary deck of 52 playing cards. If she draws any other card, she loses. How much should she pay to play if the game is fair?

$X$ - gain from each play

<table>
<thead>
<tr>
<th>$X$</th>
<th>3</th>
<th>5</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X)$</td>
<td>$\frac{8}{52}$</td>
<td>$\frac{8}{52}$</td>
<td>$\frac{36}{52}$</td>
</tr>
</tbody>
</table>

Game is fair $\iff E(X) = 0$

$E(X) = 3 \cdot \frac{8}{52} + 5 \cdot \frac{8}{52} + k \cdot \frac{36}{52} = \frac{64 + 36k}{52} = 0$

Then $k = -\frac{64}{36} = -\frac{16}{9}$

She should pay $\frac{16}{9}$.
3. A continuous random variable $X$ has the density function

$$f(x) = \begin{cases} 
  e^{-x}, & x > 0, \\
  0, & \text{elsewhere}
\end{cases}$$

Find the mean of $g(X) = e^{-2X}$.

$$E(g(X)) = \int_{-\infty}^{\infty} g(x)f(x) \, dx = \int_0^{\infty} e^{-2x} \cdot e^{-x} \, dx$$

$$= \int_0^{\infty} e^{-3x} \, dx = -\frac{1}{3} e^{-3x} \bigg|_0^{\infty} = -\frac{1}{3} (1 - 1) = \frac{1}{3}$$

4. Find the variance of $g(X) = 2X + 1$ for the $X$ given in problem 1.

$$E(2X+1) = E(2X) + E(1) = 2 E(X) + 1$$

$$= 2 \cdot 8 + 1 = 17$$

$$\text{Var}(2X+1) = 4 \text{Var}(X) = 69.24$$

or

$$\sum_x (g(x)-\mu g(x))^2 f(x) = (50.1)^2 (0.22) + (301)^2 (0.36) + 1^2 (0.28) + (-299)^2 (0.14)$$

$$-(171)^2 = 69.24$$

5. Find conditional probability $P(X = 2|Y = 3)$ from the given joint PMF

$\begin{array}{c|c|c|c|c|c}
  & x & 1 & 2 & 3 & 4 \\
  \hline
  y & \text{f} & 0.10 & 0.15 & 0.20 & 0.30 \\
  \hline
  3 & 0.5 & & & & \\
  5 & 0.10 & 0.15 & & & \\
\end{array}$

$$P(X = 2|Y = 3) = \frac{P(X = 2, Y = 3)}{P(Y = 3)} = \frac{0.2}{0.5} = 0.4$$

SCORE: 2