1. (4 points) A bus arrives every 10 minutes at a bus stop. It is assumed that the waiting time for a particular individual is a random variable with a continuous uniform distribution.

(a) What is the probability that the individual waits more than 7 minutes?
(b) What is the probability that the individual waits between 2 and 7 minutes?

\[ X \sim \text{Uniform distribution} \quad X \in [0, 10] \]

(a) \( P(X > 7) = (10 - 7) \cdot \frac{1}{10} = \frac{3}{10} \)

(b) \( P(2 < X < 7) = (7 - 2) \cdot \frac{1}{10} = \frac{1}{2} \)

2. (5 points) Assume that the percent purity of oxygen from a certain supplier follows approximately normal distribution, with mean 99.61 and standard deviation of 0.08.

(a) What percentage of the purity values would you expect to be between 99.5 and 99.7?
(b) What purity value would you expect to exceed exactly 5% of the population?

\[ X \sim \mathcal{N}(\mu, \sigma) \quad \mu = 99.61 \quad \sigma = 0.08 \]

(a) \( P(99.5 < X < 99.7) = P(-1.375 < Z < 1.125) \)

\[ = P(1.125 < Z) - P(-1.375 < Z) \approx 0.7852 \]

(b) If we want top 5% \( P(X > k) = 0.05 \)

\[ \Rightarrow P\left(Z > \frac{k - 99.61}{0.08}\right) = 0.05 \Rightarrow P\left(Z < -\frac{k - 99.61}{0.08}\right) = 0.05 \]

\[ -\frac{k - 99.61}{0.08} = -1.645 \Rightarrow k = 99.74 \]
3. (6 points) A lawyer commutes daily from his suburban home to his midtown office. The average time for a one-way trip is 24 minutes, with a standard deviation of 3.8 minutes. Assume the distribution of trip times to be normal.

(a) What is the probability that a trip will take at least 1/2 hour?

(b) If the office opens at 9am and the lawyer leave his house at 8:45am daily, what percentage of the time is he late for work?

(c) Find the probability that 2 of the next 3 trips will take at least 1/2 hour.

\[ X \sim \text{Normal}(24, 3.8) \]

\[ P(X \geq 30) = P(Z \geq 1.58) = P(Z \leq -1.58) = 0.0571 \]

\[ P(X \geq 15) = P(Z \geq -2.37) = 1 - P(Z \leq -2.37) = 0.9911 \]

The lawyer will be late for work 99.11% of the time.

(c) \[ Y \sim \text{Binomial}(3, 0.0571) \]

\[ P(Y = 2) = \binom{3}{2} (0.0571)^2 (1-0.0571) = 0.00922 \]