Math 1290 HW14 Sec 2.5: 2, 6, 10, 12, 18, 22, 26, 27, 30, 38a–c, 41

2. $f(x) = x^{-5}$ $\boxed{f'(x) = -5x^{-6}}$

6. $f(x) = x^{-e}$ $\boxed{f'(x) = -ex^{-e-1}}$

10. $s(x) = 1 - x + x^2 - x^3 + x^4$

$\boxed{s'(x) = -1 + 2x - 3x^2 + 4x^3}$

12. $p(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

$p'(x) = 1 + \frac{2x}{2} + \frac{3x^2}{6} + \frac{4x^3}{24} = \boxed{1 + x + \frac{x^2}{2} + \frac{x^3}{6}}$

18. $g(x) = 4x - x^2$ for $0 \le x \le 5$

$g'(x) = 4 - 2x$ critical points: $4 - 2x = 0$

$4 = 2x$

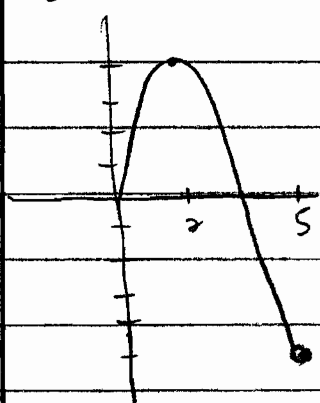
$x = 2$

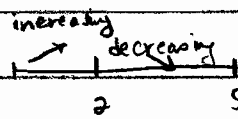
$g(2) = 4(2) - 2^2$

$= 8 - 4 = 4$

$g(0) = 0$

$g(5) = 4(5) - 5^2 = -5$





22. $f'(x) = 2x$ $f(x) = x^2 + C$

26. $f'(x) = 3x^{-4}$ $f(x) = -x^{-3} + C$

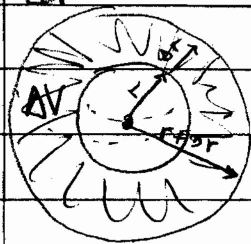
27. $A(t) = 175t^3$

$A'(t) = 525t^2 = \frac{dA}{dt}$

This is the rate of the spread of the AIDS epidemic.

30. $V(r) = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2 \ \frac{cm^3}{cm}$



The derivative is the change in volume as the radius changes. The units are $\frac{cm^3}{cm} = cm^2$

It makes sense to think of this as the volume increase as just another layer of skin (surface area) on the sphere.

38. $V_a(t) = 3t + 20 + \frac{t^2}{2}$ $\qquad V_b(t) = -1t + 40$

$V(t) = V_a(t) + V_b(t) = 3t + 20 + \frac{t^2}{2} + -1t + 40$

a) $V(t) = 2t + 60 + \frac{t^2}{2}$

b) $V_a'(t) = 3 + t \ \frac{cm^3}{day}$ $\qquad V_b'(t) = -1 \ \frac{cm^3}{day}$

c) $V'(t) = 3 + t - 1 = 2 + t \ \frac{cm^3}{day}$

$V_a'(t) + V_b'(t) = 3 + t - 1 = 2 + t \ \frac{cm^3}{day}$

41. $m(t) = 100 + 10t - \frac{22.88}{2}t^2 = 100 + 10t - 11.44t^2$ meters

$m'(t) = 10 - 22.88t$

$m'(t) = 0 \qquad 10 - 22.88t = 0$

$22.88t = 10 \Rightarrow t = \frac{10}{22.88} = 0.437$ sec

max height: $m(0.437) = 100 + 10(0.437) - 11.44(0.437)^2 = \boxed{102.185 \text{ meters}}$

$m(t) = 0 \qquad 100 + 10t - 11.44t^2 = 0$ use quadratic formula $a = -11.44$, $b = 10$, $c = 100$

$t = \frac{-10 \pm \sqrt{10^2 - 4(-11.44)(100)}}{2(-11.44)} = \boxed{3.426 \text{ sec}}$ & $\cancel{-2.552}$ sec

hits the ground

$m'(3.426) = 10 - 22.88(3.426) = \boxed{-68.38 \ \frac{\text{meters}}{\text{sec}}}$

velocity when it hits the ground.

So, speed is $|-68.38| = 68.38 \ \frac{\text{meters}}{\text{sec}}$

Math 1210 HW 15 Sec 2.6: 1, 4, 9, 12, 17, 18, 26, 36, 37

1) $f(x) = (2x+3)(-3x+2)$

$f'(x) = \underbrace{(2x+3)(-3) + (-3x+2)(2)}_{\text{ok to here.}} = -6x-9-6x+4 = \boxed{-12x+5}$

4. $s(t) = (t^2+2)(3t^2-1)$

$s'(t) = \underbrace{(t^2+2)(6t) + (3t^2-1)(2t)}_{\text{ok to here}} = 6t^3+12t+6t^3-2t = \boxed{12t^3+10t}$

9. $g(z) = \dfrac{1+z^2}{1+2z^3}$

$g'(z) = \dfrac{(1+2z^3)(2z)-(1+z^2)(6z^2)}{(1+2z^3)^2} = \dfrac{2z+4z^4-6z^2-6z^4}{1+4z^3+4z^6} = \boxed{\dfrac{-2z^4-6z^2+2z}{1+4z^3+4z^6}}$

12. $G(x) = \dfrac{(1+x)(2+x)}{3+x}$

$G'(x) = \dfrac{(3+x)[(1+x)(1)+(2+x)(1)]-(1+x)(2+x)(1)}{(3+x)^2} = \dfrac{(3+x)(3+2x)-(2+3x+x^2)}{9+6x+x^2}$

$= \dfrac{9+9x+2x^2-2-3x-x^2}{9+6x+x^2} = \boxed{\dfrac{7+6x+x^2}{9+6x+x^2}}$

17. $f(x)$ is positive and increasing for all x thus $f'(x) > 0$

$y = (f(x))^2 = f(x) \cdot f(x)$

$y' = f(x)f'(x) + f(x)f'(x)$

$y' = 2f(x)f'(x) > 0$

Thus, $(f(x))^2$ is also increasing.

18. $y = \frac{1}{f(x)}$ $\quad y' = \frac{f(x)(0) - 1f'(x)}{(f(x))^2} = \frac{-f'(x)}{(f(x))^2} < 0$

Thus, $y = \frac{1}{f(x)}$ is decreasing

26. a) $m(t) = P(t) \cdot W(t)$

$m(t) = (2\times10^6 + 2\times10^4)(80 - 0.005t^2)$

b) $m'(t) = (2\times10^6 + 2\times10^4 t)(-0.01t) + (80 - 0.005t^2)(2\times10^4)$

$m'(t) = -20000t - 200t^2 + 1.6\times10^6\ 100t^2$

$m'(t) = -300t^2 - 20000t + 1.6\times10^6$

c) $m'(t) = 0 \qquad -300t^2 - 20000t + 1.6\times10^6 = 0$

$t = \frac{-(-20000) \pm \sqrt{(-20000)^2 - 4(-300)(1.6\times10^6)}}{2(-300)} \approx \frac{20000 \pm 48166.38}{-600}$

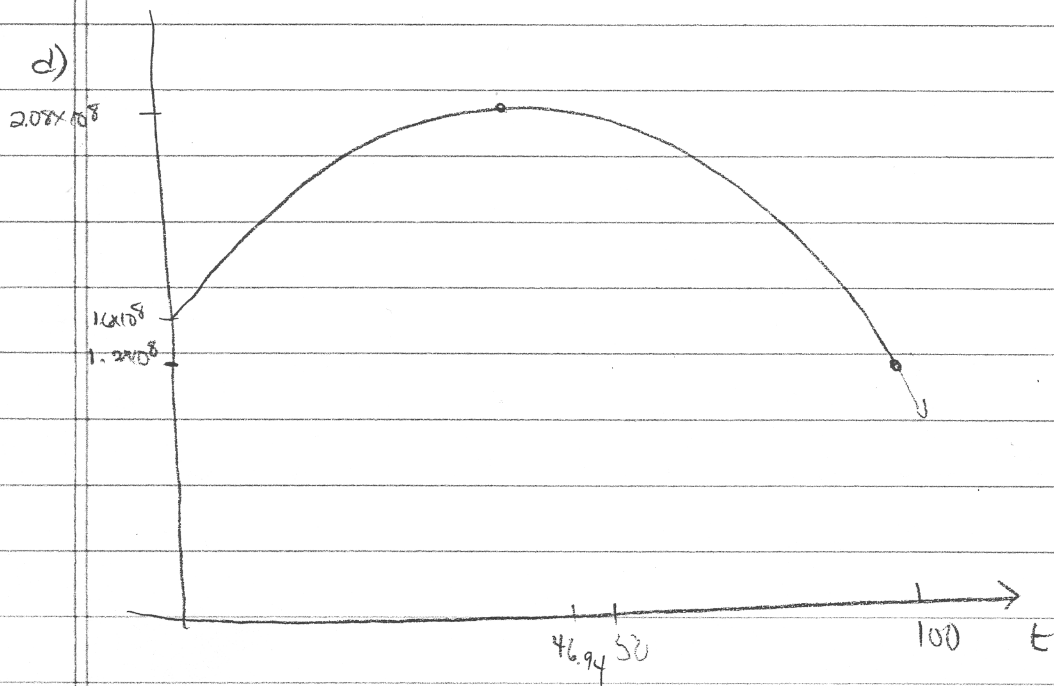
$\cancel{t = -113.6}$ $\quad \boxed{t = 46.94}$

$P(46.94) = 2\times10^6 + 2\times10^4(46.94) = 2.94\times10^6$ individuals

$W(46.94) = 80 - 0.005(46.94)^2 = 68.98$ kg

$m(46.94) = P(46.94)\,W(46.94) = 2.03\times10^8$ kg

d)



36 $m = \rho V \Rightarrow \rho = \frac{m}{v}$ a) $\rho(t) = \frac{1+t^2}{1+2t} \frac{g}{cm^3}$

b) $\rho'(t) = \frac{(1+2t)(2t) - (1+t^2)(2)}{(1+2t)^2} = \frac{2t + 4t^2 - 2 - 2t^2}{(1+2t)^2} = \frac{2t^2 + 2t - 2}{(1+2t)^2} \frac{\frac{g}{cm^3}}{day}$

c) $\rho'(t) = 0$ $\frac{2t^2+2t-2}{(1+2t)^2} = 0$ $2t^2 + 2t - 2 = 0$ $t = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$

$t^2 + t - 1 = 0$

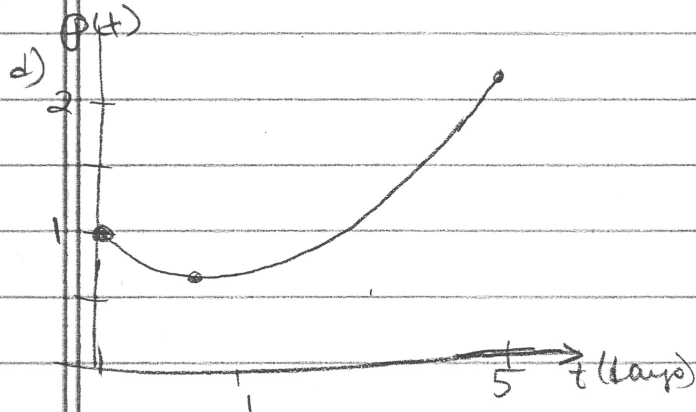
$t = \frac{-1 \pm \sqrt{5}}{2}$

$t \approx 0.618$ days or ~~$t = -1.618$~~

For $t < 0.618$ $\rho'(t) < 0$ so, $\rho(t)$ is decreasing

For $t > 0.618$ $\rho'(t) > 0$ so, $\rho(t)$ is increasing

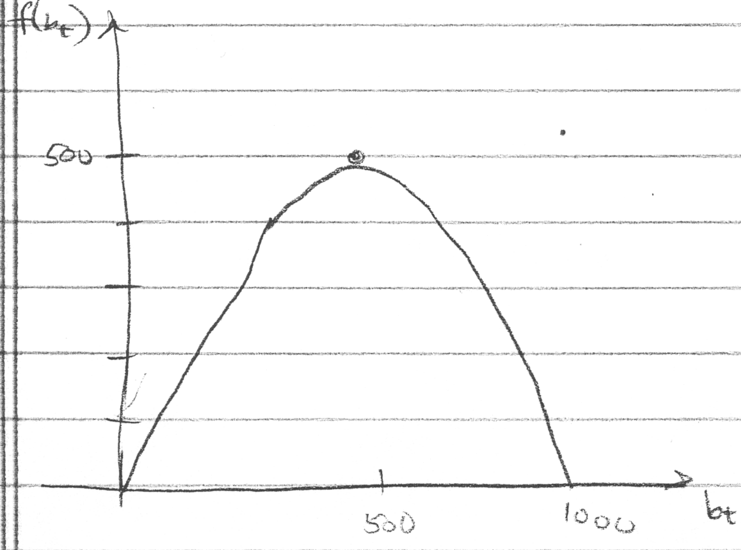
d)



t	$\rho(t)$
0	1
0.618	0.618
5	2.36

37 $f(b_t) = 2\left(1 - \frac{b_t}{1000}\right) b_t = 2b_t \left(1 - \frac{b_t}{1000}\right)$

$f'(b_t) = 2b_t\left(\frac{-1}{1000}\right) + \left(1 - \frac{b_t}{1000}\right)2 = \frac{-2b_t}{1000} + 2 - \frac{2b_t}{1000} = \frac{-4b_t}{1000} + 2$



$f'(b_t) = 0$ $-\frac{4b_t}{1000} + 2 = 0$

$2 = \frac{4b_t}{1000}$

$b_t = 500$

b_t	$f(b_t)$
0	0
500	500
1000	0