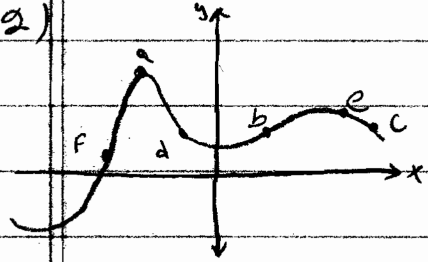
Math 1290 HW16 Sec 27: 2, 4, 6, 8, 12, 14, 16, 27, 29, 34, 42, 44

2)



answers will vary

need:

a) critical point

b) $f'(x) > 0$

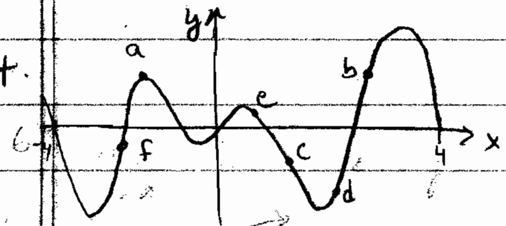
c) $f'(x) < 0$

d) $f''(x) > 0$

e) $f''(x) < 0$

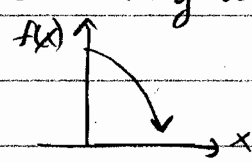
f) $f''(x) = 0$

4.



6) ans. may vary need a function w/ a positive, decreasing derivative

8) ans may vary: need a function w/ a negative, decreasing derivative



12. $g(z) = 3z^3 + 2z^2$

$g'(z) = 9z^2 + 4z$

$\boxed{g''(z) = 18z + 4}$

14. $p(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

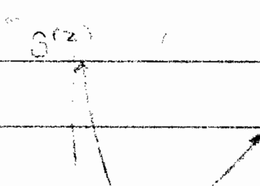
$p'(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$

$p''(x) = 1 + x + \frac{1}{2}x^2$

16. $R(s) = (1+s^2)(2+s) = 2 + 2s^2 + s + s^3$

$R'(s) = 3s^2 + 4s + 1$

$\boxed{R''(s) = 6s + 4}$



27. $y = x^9$

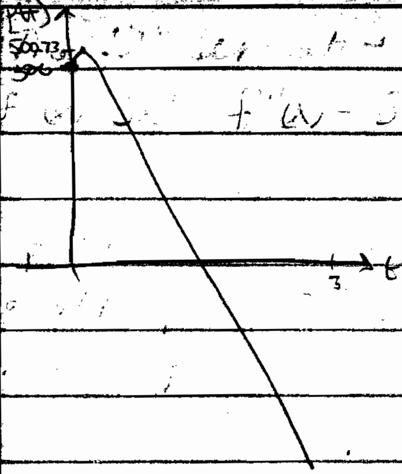
$y^{10}(x) = 0$

29. $p(x) = 7x^8 - 8x^7 - 5x^6 + 6x^5 - 4x^3$

$p^{8}(x) > 0$ because the coef is the product of positive coef.

$v(t) = p'(t) = -274t + 20$ $a(t) = p''(t) = -274 \frac{m}{s^2}$

b)



c) $p(0) = -137(0)^2 + 20(0) + 500 = \boxed{500 \text{ m}}$

$v(0) = -274(0) + 20 = 20 \frac{m}{s}$ (thrown up

The acceleration is <u>much</u> larger than on Earth.

42. most quickly ≈ 10

accelerating most quickly ≈ 8

decelerating most quickly ≈ 6

44. $f(b_t) = 2b_t\left(1 - \frac{b_t}{1000}\right)b_t = 2b_t^2\left(1 - \frac{b_t}{1000}\right)$

$f'(b_t) = 2b_t^2\left(\frac{-1}{1000}\right) + \left(1 - \frac{b_t}{1000}\right)(4b_t) = \frac{-b_t^2}{500} + 4b_t - \frac{b_t^2}{250} = \frac{-3b_t^2}{500} + 4b_t$

$f''(b_t) = \frac{-6}{500}b_t + 4$

find critical points:

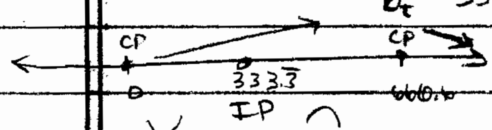
$f'(b_t) = 0$ $\frac{-3b_t^2}{500} + 4b_t = 0$ $b_t\left(\frac{-3b_t}{500} + 4\right) = 0$

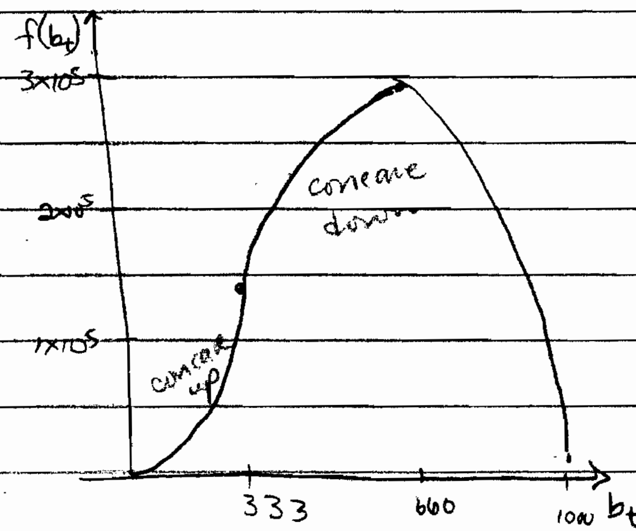
$b_t = 0$ $\frac{-3b_t}{500} + 4 = 0 \Rightarrow 4 = \frac{3b_t}{500} \Rightarrow 2000 = 3$

$b_t = 66$

$f''(b_t) = \frac{-6}{500}b_t + 4 = 0$

$b_t = 333.\overline{3}$





b_t	$f(b_t)$
0	0
333	148,146
666	296,296
1000	20

Math 1290 HW 17 Sec 2.8: 4, 6, 12, 16, 20, 27, 28, 36, 42

4. $h(x) = (2-x^2)e^x$

$h'(x) = (2-x^2)e^x + e^x(-2x) = e^x(2-x^2-2x)$

$h''(x) = e^x(-2x-2) + (2-x^2-2x)e^x = e^x(-2x-2+2-x^2-2x)$

$= \boxed{e^x(-x^2-4x)}$

6. $G(z) = \dfrac{e^z}{z^2}$

$G'(z) = \dfrac{z^2e^z - e^z(2z)}{z^4} = \dfrac{e^z(z^2-2z)}{z^4}$

$G''(z) = \dfrac{z^4(e^z(2z-2) - (z^2-2z)e^z) - e^z(z^2-2z)(4z^3)}{z^8} = \dfrac{e^z(z^4(z^2-2) - 4z^3(z^2-2z))}{z^8}$

$= \dfrac{e^z(z^6 - 2z^4 - 4z^5 + 8z^4)}{z^8} = \boxed{\dfrac{e^z(z^6 - 4z^5 + 6z^4)}{z^8}}$

-or-

$G(z) = e^z z^{-2}$

$G'(z) = e^z(-2z^{-3}) + z^{-2}e^z = e^z(-2z^{-3} + z^{-2})$

$G''(z) = e^z(6z^{-4} - 2z^{-3}) + (-2z^{-3} + z^{-2})e^z = \boxed{e^z(6z^{-4} - 4z^{-3} + z^{-2})}$

12. $f(x) = x^2 \ln x$

$f'(x) = x^2\left(\frac{1}{x}\right) + \ln(x)\cdot 2x = \boxed{x + 2x\ln x}$

$f''(x) = 1 + 2x\cdot\frac{1}{x} + \ln x \cdot 2 = 1 + 2 + 2\ln x = \boxed{3 + 2\ln x}$

16. $r(x) = \ln(x^2e^x) = \ln(x^2) + \ln(e^x) = 2\ln x + x$

$r'(x) = 2\left(\frac{1}{x}\right) + 1 = \frac{2}{x} + 1 = \boxed{2x^{-1} + 1}$

$r''(x) = \boxed{-2x^{-2}}$

20 $g(x) = (2-x)e^x \qquad -2 \le x \le 1$

$g'(x) = (2-x)e^x + e^x(-1) = e^x(2-x-1) = e^x(1-x)$

$g''(x) = e^x(-1) + (1-x)e^x = e^x(-1+1-x) = -xe^x$

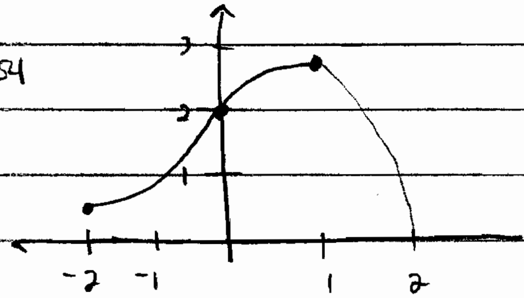
$g'(x) = 0 \qquad g''(x) = 0$

$e^x(1-x) = 0 \qquad -xe^x = 0$

$e^x = 0$ never $\qquad 1-x=0 \qquad -x=0$

$x = 1 \qquad x = 0$

x	$g(x)$
-2	$4e^{-2} \approx 0.54$
0	2
1	e



cp

0 IP $\qquad 1$

27 $h(x) = (ax+b)e^x \qquad a=1 \quad b=1$

$h(x) = (x+1)e^x$

$h'(x) = (x+1)e^x + e^x(1) = e^x(x+1+1) = e^x(x+2)$

$h''(x) = e^x(1) + (x+2)e^x = e^x(1+x+2) = e^x(x+3)$

28 $h^{10}(x) = e^x(x+11)$

36 $P(t) = 10e^t \qquad M(t) = P(t)\,m(t)$

$M(t) = 10e^t(1-t)$

$m'(t) = 10e^t(-1) + (1-t)(10e^t) = 10e^t(-1+1-t) = -10te^t$

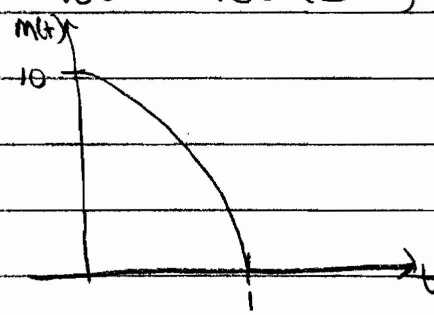
$m''(t) = -10te^t + e^t(-10) = -10te^t - 10e^t = -10e^t(t+1)$

$m'(t) = 0 \quad t = 0$

$m''(t) = 0 \quad t = 1$

cp

IP $\qquad 0$



No, The total mass, $M(t)$ is decreasing for $t > 0$.

The mass is zero for $t = 1$

42. $\frac{db}{dt} = b(t)$ $b(t) = e^t$

The differential equation says that the rate of change of the population is equivalent to the size of the population. The solution $b(t) = e^t$ says that the population grows exponentially as a function of time.

LHS: $\frac{db}{dt} = b'(t) = e^t$ ✓

RHS: $b(t) = e^t$ ✓

The solution is an increasing function (It's first derivative is positive for all t)