Math 1290 Hw 23 Sec 3.4: 2, 4, 12, 14, 16, 18, 20, 22, 30, 40

2. $e^x - 3x^2 = 0 \qquad 0 \le x \le 1$

$f(x) = e^x - 3x^2$ is the sum of 2 continuous functions & is thus continuous

$f(0) = e^0 - 3(0)^2 = 1$

$f(1) = e^1 - 3(1)^2 = e^1 - 3 \approx -0.28$

change in sign, thus by the Intermedia[te] Value Theorem there exists a solution in $[0,1]$

4. $e^x + x^2 - 2 = \cos(2\pi x) - 1$

$f(x) = e^x + x^2 - \cos(2\pi x) - 1$ is the sum of multiple continuous functions & is thus continuous.

$f(0) = e^0 + 0^2 - \cos(2\pi(0)) - 1 = -1$

$f(1) = e^1 + 1^2 - \cos(2\pi(1)) - \cancel{1} = e^1 - 1 \approx 1.71$

change in sign, thus by the IVT there exists a solution in $[0,1]$.

12. $f(x) = x^2 \qquad x = 0$ to $x = 2$

* $f(x)$ is continuous on $[0,2]$

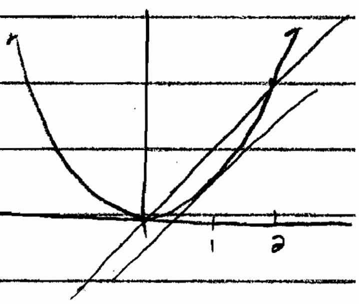
* $f(x)$ is diff on $(0,2)$

$F'(x) = 2x$

$f'(c) = \dfrac{f(b) - f(a)}{b - a} = \dfrac{f(2) - f(0)}{2 - 0} = \dfrac{4 - 0}{2} = 2$

$f'(c) = 2c = 2$

$\boxed{c = 1}$



14. $f(x) = \sqrt{x} \qquad x = 0$ to $x = 2$

* $f(x)$ is continuous on $[0,2]$

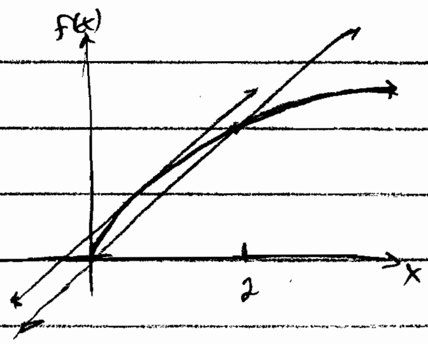
$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f'(c) = \dfrac{f(b) - f(a)}{b - a} = \dfrac{\sqrt{2} - \sqrt{0}}{2 - 0} = \dfrac{\sqrt{2}}{2}$

$f'(c) = \dfrac{1}{2\sqrt{c}} = \dfrac{\sqrt{2}}{2} \implies 2\sqrt{2}\sqrt{c} = 2$

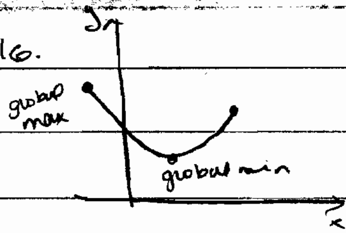
$\sqrt{c} = \dfrac{2}{2\sqrt{2}}$

$c = \frac{1}{2}$

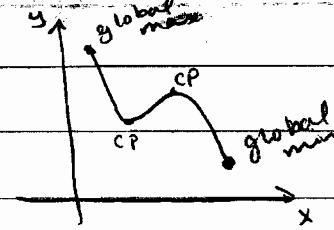


16-20 have many possible answers

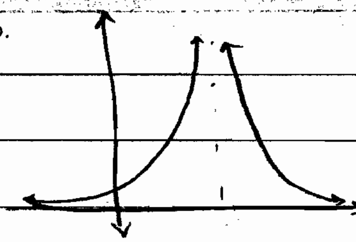
16.



18.

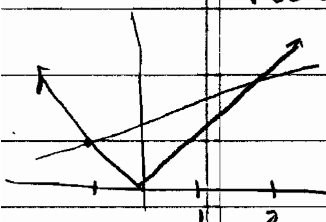


20.



22 $g(x) = |x|$ $g(x)$ is continuous for all real numbers

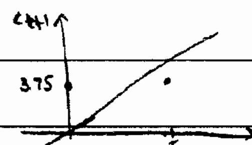
Yes, it satisfies the conditions of the Intermediate Value Theorem.



$$\frac{g(-1) - g(2)}{-1-2} = \frac{1-2}{-3} = \frac{-1}{-3} = \frac{1}{3}$$

But the slopes of the tangent lines are -1 for $x < 0$ and 1 for $x > 0$. There is never a value of x for which $f'(x) = \frac{1}{3}$

However, this does not contradict the Mean Value Theorem because its conditions are not satisfied. Specifically, $f(x)$ is not differentiable at $x=0$



30. $c_{t+1} = 0.25e^{-3c_t}c_t + 0.750$ where $\delta = 5$

$f(c_t) = 0.25e^{-3c_t}c_t + 3.75$ * note $f(c_T)$ is a continuous function

$f(0) = 0.25e^{-3(0)}(0) + 3.75 = 3.75$ here $c_{t+1} > c_t$

$f(5) = 0.25e^{-3(5)}(5) + 3.75 = 3.75$ here $c_{t+1} < c_t$

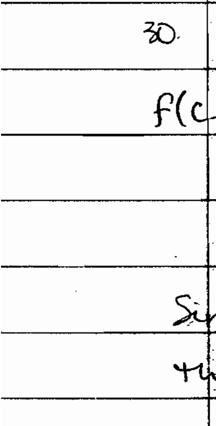
Since equilibrium points occur where $c_t = c_{t+1}$, then between $c_t = 0$ & $c_t = \delta = 5$ the updating function must cross the diagonal.

—or— $c^* = 0.25e^{-3c^*}c^* + 3.75 \rightarrow f(c^*) = 0.25e^{-3c^*}c^* + 3.75 - c^*$ $f(c^*)$ is continuous and

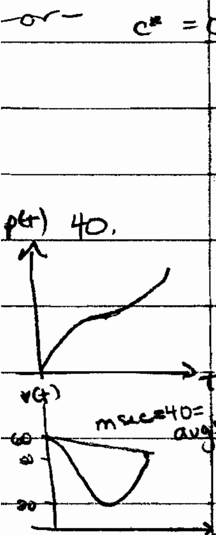
$f(0) = 0.25e^{-3(0)}(0) + 3.75 - 0 = 3.75$ and $f(5) = 0.25e^{-3(5)}(5) + 3.75 - 5 = -1.25$

Since there is a change of sign, by IVT there must be a solution between $c^* = 0$ & $c^* = 5$

40. According to the Mean Value Theorem the speed (instantaneous velocity) at some point during the hour must be equal to the average velocity (40 mph)



According to the Intermediate Value Theorem the instantaneous velocity must take on every velocity from 20 to 60 mph during the hour.



Sec 3.5

Math 1290 HW24: 2, 4, 8, 10, 14, 18, 22, 24, 26, 32, 34, 37-39

2. $\lim_{x\to\infty} \ln\left(\frac{x}{5}\right) = \infty$

4. $\lim_{x\to\infty} 1-e^{-4x} = 1$ because $\lim_{x\to\infty} e^{-4x} = 0$

8. $\lim_{x\to\infty} (x^2+2)^{0.25} = \infty$

10.

x	x^3	$1000x$
1	1	1000
10	1000	10000
100	1,000,000	100,00

$x^3 \to \infty$ faster than $1000x$ as $x\to\infty$

Rule: $ax^n \to \infty$ faster for larger n

The order switches as x gets larger

$x^3 = 1000x$

$x^2 = 1000$ $x = 31.62$ So, for $\boxed{x > 31.61}$

14.

x	$10x^{0.1}$	$x^{0.5}$
1	10	1
10	12.6	3.16
100	15.8	10

$x^{0.5} \to \infty$ faster than $10x^{0.1}$ as $x\to\infty$

Rule: $ax^n \to \infty$ faster for larger n.

The order switches as x gets larger

$\frac{10x^{0.1}}{x^{0.1}} = \frac{x^{0.5}}{x^{0.1}} \Rightarrow (10)^{\frac{1}{0.4}} (x^{0.4})^{\frac{1}{0.4}}$

$x = 316.2$ So, $\boxed{x > 316.2}$

18.

x	$x^{-0.1}$	$25x^{-0.2}$
1	1	25
10	0.79	15.7
100	0.63	9.95

$25x^{-0.2} \to 0$ faster than $x^{-0.1}$ as $x\to\infty$

Rule: $ax^{-n} \to 0$ faster for larger n

The order switches as x gets larger.

$\frac{x^{-0.1}}{x^{-0.2}} = \frac{25x^{-0.2}}{x^{-0.2}}$

$x^{0.1} = 25$ So, for $x > 9.5\times10^{13}$

$x = (25)^{\frac{1}{0.1}}$

$x = 9.5\times10^{13}$

22. $\lim_{c\to\infty} \frac{Ac}{\ln(1+c)} = \infty$

24. $\lim_{c\to\infty} \frac{c^2}{1+10c} = \infty$

26. $\alpha(c) = \frac{5c}{1+c}$

$\alpha'(c) = \frac{(1+c)(5)-(5c)(1)}{(1+c)^2} = \frac{5+5c-5c}{(1+c)^2} = \frac{5}{(1+c)^2}$

$\alpha'(0) = \frac{5}{(1+0)^2} = 5$ $\quad \lim_{c\to\infty} \alpha'(c) = \lim_{c\to\infty} \frac{5}{(1+c)^2} = 0$

Yes, the curve starts with a positive rate of change that decreases (levels off) to zero.

32. $b_{t+1} = rb_t \quad b_t = b_0 r^t$ $\qquad 10^8(1.5)^t = 10^{10}$

$b_{t+1} = 1.5b_t \quad b_t = 10^8(1.5)^t$ $\qquad (1.5)^t = \frac{10^{10}}{10^8} = 10^2$

$\lim_{t\to\infty} b_t = \lim_{t\to\infty} 10^8(1.5)^t = \boxed{\infty}$ $\qquad t\ln(1.5) = \ln(10^2)$

$\boxed{t = 11.36}$

34. $b_{t+1} = 0.9b_t \quad b_t = 10^8(0.9)^t$ $\qquad 10^8(0.9)^t = 10^3$

$\lim_{t\to\infty} b_t = \lim_{t\to\infty} 10^8(0.9)^t = \boxed{0}$ $\qquad (0.9)^t = \frac{10^3}{10^8} = 10^{-5}$

$t\ln(0.9) = \ln(10^{-5})$

$\boxed{t = 109.27}$

37. $m_{t+1} = 0.5m_t + 1$

$m^* = 0.5m^* + 1$

$0.5m^* = 1 \quad m^* = 2$

38. $\lim_{t\to\infty} m_t = \lim_{t\to\infty} 2 + (0.5)^t 3 = 2$

39. 1% of $2 = 0.02$

$2.02 = 2 + (0.5)^t 3$

$0.02 = (0.5)^t(3)$

$\frac{0.02}{3} = (0.5)^t$

$\ln\left(\frac{0.02}{3}\right) = t\ln(0.5)$

$t = \frac{\ln\left(\frac{0.02}{3}\right)}{\ln(0.5)}$

$\boxed{t \approx 7.2}$