Math 1290 HW25 Sec 3.6: 4, 6, 10, 12, 14, 18, 21, 22, 28

4. $F(x) = 1 + 2x + 3e^{x}$

$F_0(x) = 3e^{x} \quad F_\infty(x) = 3e^{x}$

6. $G(c) = e^{-4c} + \frac{5}{c^2} + \frac{3}{c^5} + 10e^{-3c}$

$G_0(c) = \frac{3}{c^5} \quad G_\infty(c) = \frac{5}{c^2}$

10. $x \to \infty$ faster than $(\ln(x))^2$ as $x \to \infty$

$$\lim_{x\to\infty} \frac{x}{(\ln(x))^2} \underset{\frac{\infty}{\infty}}{\overset{LH}{=}} \lim_{x\to\infty} \frac{1}{2\ln(x)\cdot\frac{1}{x}} = \lim_{x\to\infty} \frac{x}{2\ln x} \underset{\frac{\infty}{\infty}}{\overset{LH}{=}} \lim_{x\to\infty} \frac{1}{2\frac{1}{x}} = \lim_{x\to\infty} \frac{x}{2} = \infty$$

Yes, $x \to \infty$ faster.

12. $30x^{-0.1} \to 0$ faster than $\frac{1}{\ln x}$ as $x \to \infty$

$$\lim_{x\to\infty} \frac{\frac{1}{\ln x}}{30x^{-0.1}} = \lim_{x\to\infty} \frac{x^{0.1}}{30\ln x} \underset{\frac{\infty}{\infty}}{\overset{LH}{=}} \lim_{x\to\infty} \frac{0.1x^{-0.9}}{30\,\frac{1}{x}} = \lim_{x\to\infty} \frac{0.1x^{-0.9}x}{30} = \lim_{x\to\infty} \frac{0.1x^{0.1}}{30} = \infty$$

∴ $30x^{-0.1}$ does approach 0 faster as $x \to \infty$.

14. It's not clear from the functions which approaches ∞ faster.

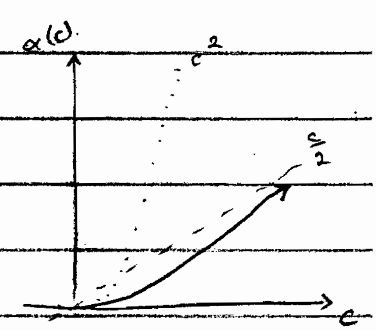
$$\lim_{x\to 0} \frac{x^{-1}}{\frac{1}{e^x - 1}} = \lim_{x\to 0} \frac{e^x - 1}{x} \underset{\frac{0}{0}}{\overset{LH}{=}} \lim_{x\to\infty} \frac{e^x}{1} = 1$$

∴ The functions approach ∞ at the same rate.

18. $\alpha(c) = \frac{c^2}{1+2c}$

	$c \to 0$	$c \to \infty$
numerator leading behavior:	c^2	c^2
denominator leading behavior:	1	$2c$

$\alpha_0(c) = \frac{c^2}{1} = c^2 \qquad \alpha_\infty(c) = \frac{c^2}{2c} = \frac{c}{2}$



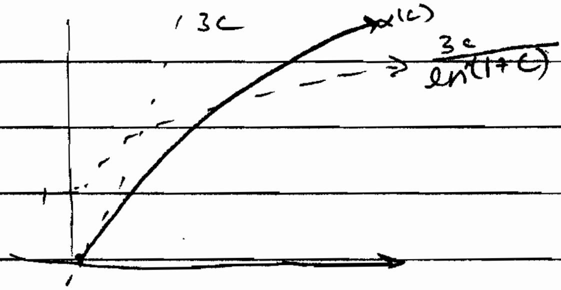
$$\lim_{c\to 0} \frac{c^2}{1+2c} = 0 \qquad \lim_{c\to\infty} \frac{c^2}{1+2c} \underset{\frac{\infty}{\infty}}{\overset{LH}{=}} \lim_{c\to\infty} \frac{2c}{2} = \infty$$

21. $\alpha(c) = \dfrac{3c}{1+\ln(1+c)}$

	$c \to 0$	$c \to \infty$
numerator leading behavior:	$3c$	$3c$
denominator leading behavior:	1	$\ln(1+c)$

$\alpha_0(c) = \dfrac{3c}{1} = 3c$ $\qquad \alpha_\infty(c) = \dfrac{3c}{\ln(1+c)}$

$\lim\limits_{c\to 0} \dfrac{3c}{1+\ln(1+c)} = 0 \qquad \lim\limits_{c\to\infty} \dfrac{3c}{1+\ln(1+c)} = \lim\limits_{c\to\infty} \dfrac{3}{\frac{1}{1+c}} = \lim\limits_{c\to\infty} 3(1+c) = \infty$

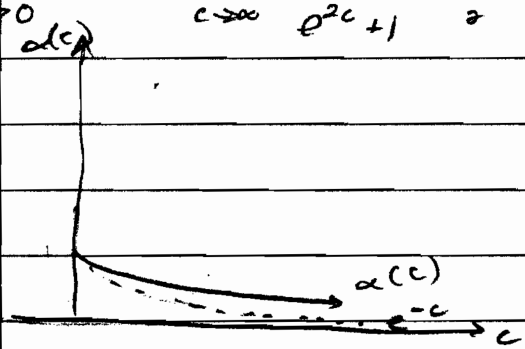


22. $\alpha(c) = \dfrac{e^c+1}{e^{2c}+1}$

	$c \to 0$	$c \to \infty$
numerator leading behavior	none	e^c
denominator leading behavior	none	e^{2c}

$\alpha_0(c) = \alpha(c)$ $\qquad \alpha_\infty(c) = \dfrac{e^c}{e^{2c}} = \dfrac{1}{e^c} = e^{-c}$

$\lim\limits_{c\to 0} \alpha(c) = \lim\limits_{c\to\infty} \dfrac{e^c+1}{e^{2c}+1} = \dfrac{2}{2} = 1 \qquad \lim\limits_{c\to\infty} \alpha(c) = \lim\limits_{c\to\infty} \dfrac{e^c+1}{e^{2c}+1} \overset{H}{\underset{\infty/\infty}{=}} \lim\limits_{c\to\infty} \dfrac{e^c}{2e^{2c}} = \lim\limits_{c\to\infty} \dfrac{1}{2e^c} = 0$



28. $\alpha(c) = \dfrac{c}{5+c}$

	$c \to 0$	$c \to \infty$
numerator leading behavior:	c	c
denominator leading behavior:	5	c

$\alpha_0(c) = \dfrac{c}{5}$ $\qquad \alpha_\infty(0) = \dfrac{c}{c} = 1$

$\lim\limits_{c\to 0} \dfrac{c}{5+c} = 0 \qquad \lim\limits_{c\to\infty} \dfrac{c}{5+c} \overset{H}{\underset{\infty/\infty}{=}} \lim\limits_{c\to\infty} \dfrac{1}{1} = 1$