Math 1290 HW26 Sec 3.7: 2, 3, 6, 9, 12, 18, 19

2. $(3.03)^2$ base point: $a=3$ $f(x)=x^2$ $f'(x)=2x$

Tangent line: $\hat{f}(x)=f(a)+f'(a)(x-a) = 9+6(x-3) = \boxed{6x-9}$

$(3.03)^2 \approx 6(3.03)-9 = \boxed{9.18}$

Secant line: using $x=2$ & $x=3$ (ok to use other points)

$m_{sec} = \frac{\Delta f}{\Delta x} = \frac{9-4}{3-2} = \frac{5}{1} = 5$ $y=5x+b$

$9=5(3)+b \Rightarrow b=-6$ $\boxed{\hat{f}_s(x)=5x-6}$

$(3.03)^2 \approx 5(3.03)-6 = \boxed{9.15}$

Actual: $(3.03)^2 = 9.1809$ The tangent line approximation is better, but both approximations are close.

3. $\sqrt{4.01}$ base point: $a=4$ $f(x)=\sqrt{x}$ $f'(x)=\frac{1}{2\sqrt{x}}$

Tangent line: $\hat{f}(x)=f(a)+f'(a)(x-a) = 2+\frac{1}{4}(x-4) = 1+\frac{1}{4}x$

$\sqrt{4.01} \approx 1+\frac{1}{4}(4.01) = \boxed{2.0025}$

Secant line: using $x=4$ & $x=5$ (again ok to use other points)

$m_{sec} = \frac{\Delta f}{\Delta x} = \frac{\sqrt{5}-\sqrt{4}}{5-4} = \frac{\sqrt{5}-2}{1} = \sqrt{5}-2$ $y=(\sqrt{5}-2)x+b$

$2=(\sqrt{5}-2)(4)+b$

$b \approx 1.0557$

$\boxed{\hat{f}_s(x)=(\sqrt{5}-2)x+1.0557 \quad \hat{f}_s(x) \approx 0.24x+1.0557}$

$\sqrt{4.01} \approx 0.24(4.01)+1.0557 = \boxed{2.0023}$

Actual: $\sqrt{4.01} = 2.002498439$ The tangent line approximation is better, but both approximations are close.

6. $\cos(-0.02)$ base point: $a=0$ $f(x)=\cos x$ $f'(x)=-\sin x$

Tangent line $\hat{f}(x)=f(a)+f'(a)(x-a) = 1+0(x-0) = \boxed{1}$

$\cos(-0.02) \approx 1$

Secant line: using $x=0$ $x=\pi$

$m_{sec} = \frac{\Delta f}{\Delta x} = \frac{\cos(\pi)-\cos(0)}{\pi-0} = \frac{-1-1}{\pi} = \frac{-2}{\pi}$ $y=\frac{-2}{\pi}x+b$

$1 = \frac{-2}{\pi}(0)+b \Rightarrow b=1$ $\boxed{\hat{f}_s(x)=\frac{-2}{\pi}x+1}$

$\cos(-0.02) \approx \frac{-2}{\pi}(-0.02)+1 = 1.0127$

Actual: $\cos(-0.02) = 0.9998$

Both approximations are over estimates but the tangent line approximation is closer.

9. $f(x)=\sqrt{x}$ $f'(x)=\frac{1}{2\sqrt{x}}=\frac{1}{2}x^{-\frac{1}{2}}$ $f''(x)=\frac{-1}{4}x^{-\frac{3}{2}}=\frac{-1}{4x^{3/2}}$ base point $a=4$

$\hat{f}(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2}(x-a)^2$

$=\sqrt{4}+\frac{1}{2\sqrt{4}}(x-4)+\frac{-1}{4(4)^{3/2}}(x-4)^2=2+\frac{1}{4}(x-4)-\frac{1}{32}(x-4)^2$

$\sqrt{4.01}\approx 2+\frac{1}{4}(4.01-4)-\frac{1}{32}(4.01-4)^2=2.002497$

actual $\sqrt{4.01}=2.0025$ So the quadratic approximation very closely matches the actual value

12. $f(x)=\cos x$ $f'(x)=-\sin x$ $f''(x)=-\cos x$ base point $a=0$

$f(0)=\cos(0)=1$ $f'(0)=-\sin(0)=0$ $f''(0)=-\cos(0)=-1$

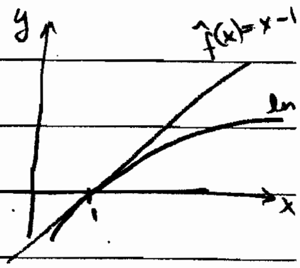
$\hat{f}(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2}(x-a)^2=1+0(x-0)+\frac{-1}{2}(x-0)^2=\boxed{1-\frac{1}{2}x^2}$

$\cos(-0.02)\approx 1-\frac{1}{2}(0.02)^2=0.9998$.

This is very close to the actual value of $\cos(-0.02)$

18. $\ln(1.1)$ $\ln(0.9)$ $f(x)=\ln(x)$ $f'(x)=\frac{1}{x}$ basepoint: $a=1$

$\hat{f}(x)=f(a)+f'(a)(x-a)=\ln(1)+1(x-1)=\boxed{x-1}$



$\ln(1.1)\approx 1.1-1=0.1$ actual $\ln(1.1)=0.0953$

$\ln(0.9)\approx 0.9-1=-0.1$ $\ln(0.9)=-0.1053$

Both approximations are over estimates because the tangent line lies above the graph.

19. $(1.1)^2$ $(0.9)^2$ $f(x)=x^2$ $f'(x)=2x$ base point: $a=1$

$\hat{f}(x)=f(a)+f'(a)(x-a)=f(1)+f'(1)(x-1)=1+2(x-1)=\boxed{2x-1}$

$(1.1)^2\approx 2(1.1)-1=1.2$ actual: $(1.1)^2=1.21$

$(0.9)^2\approx 2(0.9)-1=0.8$ $(0.9)^2=0.81$

Both approximations are under estimates because the tangent line lies below the graph.