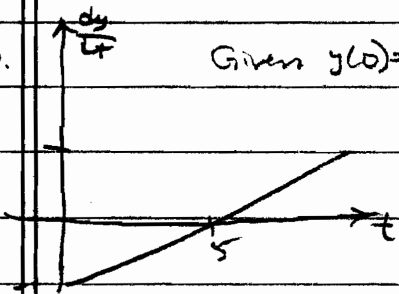
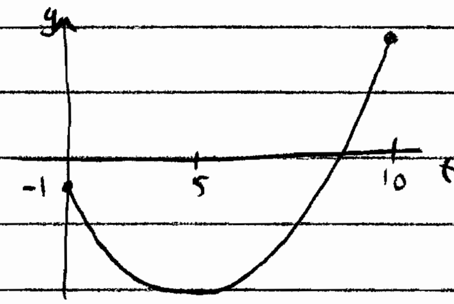
Math 1290 HW 28 Sec 4.1 : 2, 3, 6, 7, 10, 12, 14, 16, 17, 18, 22, 24

2. $\frac{dy}{dt} = 2y$ autonomous

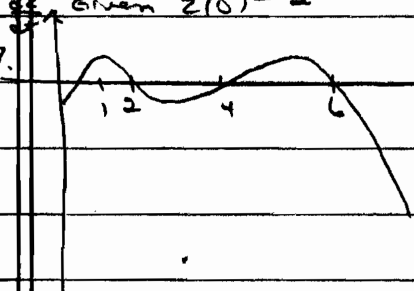
3. $\frac{dw}{dt} = \frac{2}{1+t}$ pure time

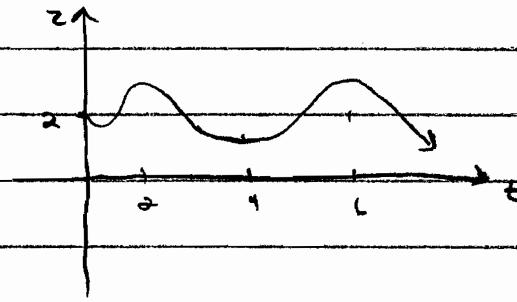
6. Given $y(0) = -1$





7. Given $z(0) = 2$





10. $\frac{dw}{dt} = \frac{2}{1+t}$ $w(t) = 2\ln(1+t) + 3$

Initial condition

LHS: $\frac{dw}{dt} = w'(t) = \frac{2}{1+t}(1) + 0 = \frac{2}{1+t}$ ✓ $w(0) = 2\ln(1+0) + 3 = 3$ ✓

RHS: $\frac{2}{1+t}$ ✓

$\therefore$ yes, $w(t)$ is a valid solution

12. $\frac{dz}{dt} = 2\sqrt{z}$ $z(t) = (t+2)^2$ Initial condition

LHS: $\frac{dz}{dt} = z'(t) = 2(t+2)'(1) = 2(t+2)$ ✓ $z(0) = (0+2)^2 = 2^2 = 4$

RHS: $2\sqrt{z} = 2\sqrt{(t+2)^2} = 2(t+2)$ ✓ $z(0) = 4$

$\therefore$ yes, $z(t)$ is a valid solution.

14. $\frac{dw}{dt} = \frac{2}{1+t}$ $w(0) = 3$ $\Delta t = 1$

one step $\hat{w}(1) = w(0) + w'(0)\Delta t$

$= 3 + \frac{2}{1+0}(1) = 3+2 = \boxed{5}$

Two step $\Delta t = 0.5$

$\hat{w}(0.5) = w(0) + w'(0)\Delta t$

$= 3 + \frac{2}{1+0}(0.5) = 3 + 2(0.5) = 4$

$\hat{w}(1) = \hat{w}(0.5) + w'(0.5)\Delta t$

$= 4 + \frac{2}{1+0.5}(0.5) = 4 + 0.\bar{6} = 4.\bar{6}$

actual $w(t) = 2\ln(1+t) + 3$

$w(1) = 2\ln(1+1) + 3 \approx 4.386$

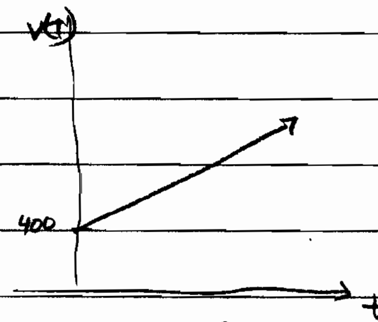
So, using a smaller time step of $\Delta t = 0.5$ yields a better approximation.

16. $V(0) = 400\ \mu m^3$ $\frac{dV}{dt} = 3$

$V(t) = 3t + C$

$V(0) = 3(0) + C = 400 \Rightarrow C = 400$

$\boxed{V(t) = 3t + 400}$



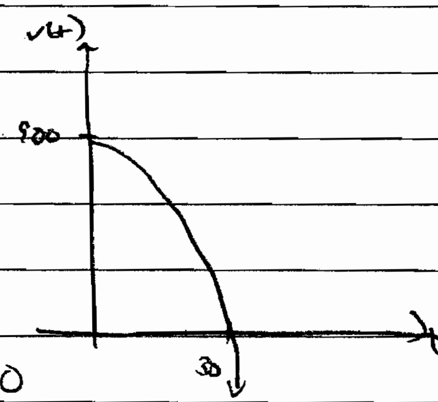
mathematically this is valid, but the cell could not get infinitely large.

17. $V(0) = 900\ \mu m^3$ $\frac{dV}{dt} = -2t\ \frac{\mu m^3}{sec}$

$V(t) = -t^2 + C$

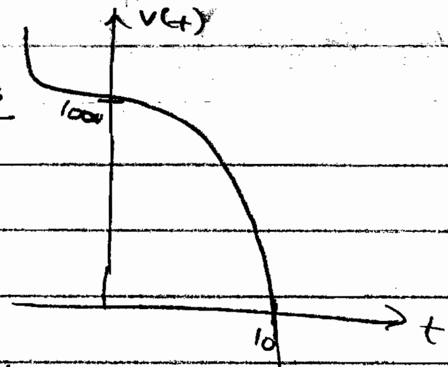
$V(0) = -(0)^2 + C = 900 \Rightarrow C = 900$

$V(t) = -t^2 + 900$



The solution is valid for $t \le \sqrt{900} = 30$ because the cell volume can not be negative.

18. $V(0) = 1000\ \mu m^3$ $\quad \frac{dV}{dt} = -3t^2 \frac{\mu m^3}{sec}$



$V(t) = -t^3 + C$

$V(0) = -0^3 + C = 1000 \Rightarrow C = 1000$

$V(t) = -t^3 + 1000$

The solution is valid for $t \leq (1000)^{\frac{1}{3}} = 10$ because for $t > 10$ the cell volume would be negative

22 $V(0) = 400 \quad V'(t) = \frac{dV}{dt} = 3 \quad \Delta t = 5$

$\hat{V}(5) = V(0) + V'(0)\Delta t = 400 + 3(5) = 415$

$\hat{V}(10) = \hat{V}(5) + V'(5)\Delta t = 415 + 3(5) = 430$

$\hat{V}(15) = \hat{V}(10) + V'(10)\Delta t = 430 + 3(5) = 445$

$\hat{V}(20) = \hat{V}(15) + V'(15)\Delta t = 445 + 3(5) = 460$

$\hat{V}(25) = \hat{V}(20) + V'(20)\Delta t = 460 + 3(5) = 475$

$\hat{V}(30) = \hat{V}(25) + V'(25)\Delta t = 475 + 3(5) = 490$

From #16 $\quad V(t) = 3t + 400 \qquad V(30) = 3(30) + 400 = 490$

Euler's method matches the exact result

24 $V(0) = 1000\ \mu m^3 \quad V'(t) = \frac{dV}{dt} = -3t^2 \quad \Delta t = 2$

$\hat{V}(2) = V(0) + V'(0)\Delta t = 1000 + -3(0)^2(2) = 1000$

$\hat{V}(4) = \hat{V}(2) + V'(2)\Delta t = 1000 + -3(2)^2(2) = 976$

$\hat{V}(6) = \hat{V}(4) + V'(4)\Delta t = 976 + -3(4)^2(2) = 880$

$\hat{V}(8) = \hat{V}(6) + V'(6)\Delta t = 880 + -3(6)^2(2) = 664$

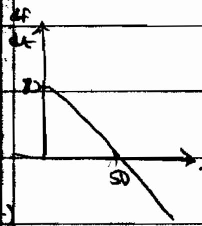
$\hat{V}(10) = \hat{V}(8) + V'(8)\Delta t = 664 + -3(8)^2(2) = 280$

From #18 $\quad V(t) = -t^3 + 1000$

$V(10) = -(10)^3 + 1000 = 0$

Euler's method over approximates the actual solution found in 18.

Math 1290 HW 29 Sec 4.2: 2, 4, 6, 8, 12, 18, 20, 26, 28

2. 

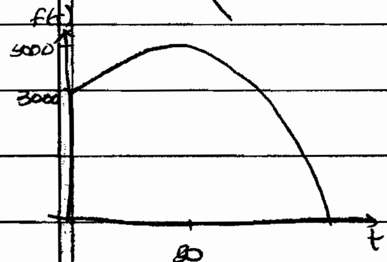
$m = \frac{-80}{50} = \frac{-8}{5}$

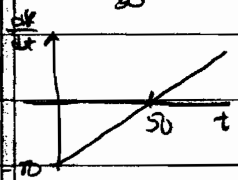
$\frac{df}{dt} = \frac{-8}{5}t + 80$

$f(t) = \frac{-8}{5}\frac{t^2}{2} + 80t + C$ Initial cond: (50,5000)

$f(50) = \frac{-8}{10}(50)^2 + 80(50) + C = 5000 \Rightarrow C = 3000$

$\boxed{f(t) = \frac{-8}{10}t^2 + 80t + 3000}$



4. 

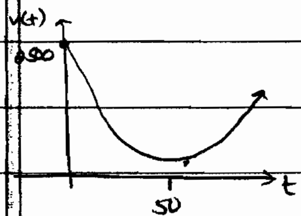
$m = \frac{80}{50} = \frac{8}{5}$

$\frac{dv}{dt} = \frac{8}{5}t - 80$

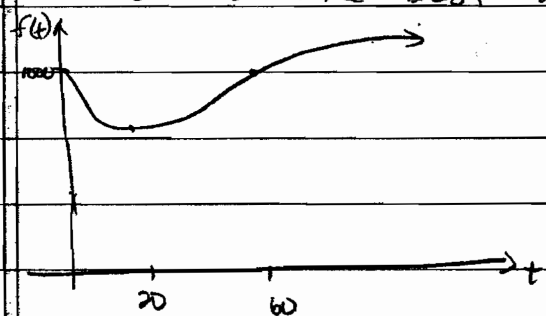
$v(t) = \frac{8}{5}\frac{t^2}{2} - 80t + C$ Initial cond: (0,2500)

$v(0) = \frac{8}{5}\frac{(0)^2}{2} - 80(0) + C = 2500 \Rightarrow C = 2500$

$v(t) = \frac{8}{10}t^2 - 80t + 2500 = \frac{4}{5}t^2 - 80t + 2500$



6. Because the diff eq. is <u>not</u> a straight line, we will not be able to get an exact formula for its solution. Just a sketch is the best we can do.



8. $\int 10t^9 + 6t^5\,dt = \boxed{t^{10} + t^6 + C}$

12. $\int 3z^{\frac{3}{7}}\,dt = 3\int z^{\frac{3}{7}}\,dt = 3 \cdot \frac{1}{\frac{10}{7}} z^{\frac{3}{7}+\frac{7}{7}} = \frac{21}{10} z^{\frac{10}{7}} + C$

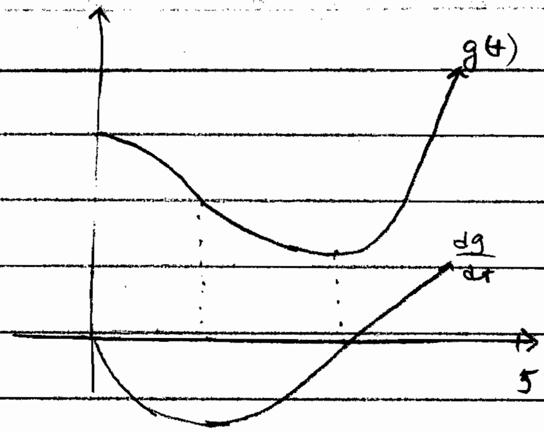
18. $\frac{dg}{dt} = -3t + t^2 \qquad g(0) = 10$

$g(t) = -\frac{3}{2}t^2 + \frac{1}{3}t^3 + C$

$g(0) = -\frac{3}{2}(0)^2 + \frac{1}{3}(0)^3 + C = 10 \Rightarrow C = 10$

$g(t) = -\frac{3}{2}t^2 + \frac{1}{3}t^3 + 10$

graph required

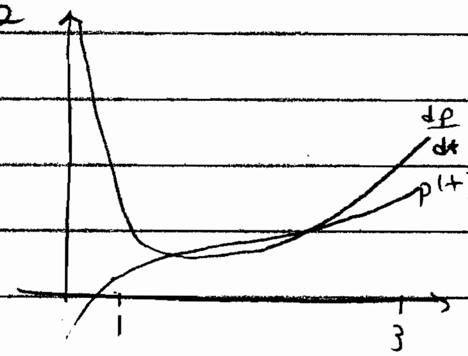


20. $\frac{dp}{dt} = 5t^3 + \frac{5}{t^2} = 5t^3 + 5t^{-2} \qquad p(1) = 12$

$p(t) = \frac{5}{4}t^4 - \frac{5}{t} + C$

$p(1) = \frac{5}{4}(1)^4 - \frac{5}{1} + C = 12 \Rightarrow C = 15.75$

$p(t) = \frac{5}{4}t^4 - \frac{5}{t} + 15.75$



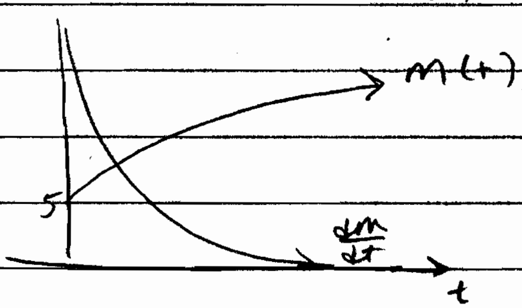
26. $n = \frac{-1}{2}, \quad \alpha = 2 \qquad \frac{dM}{dt} = \alpha t^n \qquad \frac{dM}{dt} = 2t^{-\frac{1}{2}} = \frac{2}{\sqrt{t}}$

a) units for α: $\frac{\text{grams}}{\sqrt{\text{day}}}$ because $\frac{dM}{dt} = \frac{\text{grams}}{\text{day}}$

b) $M(t) = \int \frac{dM}{dt}\,dt = \int 2t^{-\frac{1}{2}}\,dt = 4t^{\frac{1}{2}} + C$

$M(0) = 5 \qquad M(0) = 4(0)^{\frac{1}{2}} + C = 5 \Rightarrow C = 5 \qquad \boxed{M(t) = 4t^{\frac{1}{2}} + 5}$

c)



d) The mass increases at a decreasing rate.

28. See Ex. 4.27 p355

$a = 1.62\ m/s^2 \qquad v(0) = 5\ m/s \qquad h(0) = 100\,m$

a) $a(t) = \frac{dv}{dt} = -1.62\ \frac{m}{s^2}$

$v(t) = \int a(t)\,dt = -1.62t + C$

$v(0) = 5 \qquad v(0) = -1.62(0) + C = 5 \Rightarrow C = 5$

$v(t) = -1.62t + 5$

$h(t) = \int v(t)\,dt = \int -1.62t + 5\,dt = \frac{-1.62}{2}t^2 + 5t + C$

$h(0) = 100 \Rightarrow h(0) = -0.81(0)^2 + 5(0) + C = 100 \Rightarrow C = 100$

$h(t) = -0.81t^2 + 5t + 100$

b) $v(t) = 0$ (max height) $\qquad -1.62t + 5 = 0$

$t = \frac{-5}{-1.62} = \boxed{3.086\ \text{sec}}$

$h(3.086) = -0.81(3.086)^2 + 5(3.086) + 100 = \boxed{107.71\ m}$

c) $h(t) = 100 \qquad -0.81t^2 + 5t + 100 = 100$

$-0.81t^2 + 5t = 0$

$t(-0.81t + 5) = 0$

$t = 0 \qquad -0.81t = -5$

$t = 6.17\ \text{sec}$

$v(6.17) = -1.62(6.17) + 5 = -5\ m/s$

d) $h(t) = 0 \qquad -0.81t^2 + 5t + 100 = 0$

use quadratic formula, Newtons method, or calculator to find that it takes 14.62 sec to hit the ground

$v(14.62) = -1.62(14.62) + 5 = -18.68\ m/s$

e)