Math 1290 HW 30 Sec 4.3: 2, 4, 6, 8, 9, 10, 12, 14, 20, 22, 24, 32, 36, 38

2. $\int 3e^x + 2x^3\,dx = 3e^x + \frac{2}{4}x^4 + C = 3e^x + \frac{1}{2}x^4 + C$

4. $\int \frac{2}{t} + \frac{t}{2}\,dt = \int 2t^{-1} + \frac{1}{2}t\,dt = 2\ln|t| + \frac{1}{4}t^2 + C$

6. $\int x^2 - 20\sin(x)\,dx = \frac{1}{3}x^3 + 20\cos(x) + C$

8. $\int \cos(2\pi(x-2))\,dx = \int \cos(u)\frac{1}{2\pi}\,du = \frac{1}{2\pi}\int \cos(u)\,du$

let $u = 2\pi(x-2)$

$du = 2\pi\,dx$

$\frac{1}{2\pi}du = dx$

$= \frac{1}{2\pi}\sin(u) + C$

$= \frac{1}{2\pi}\sin(2\pi(x-2)) + C$

9. $\int (1 + \frac{t}{2})^4\,dt = \int u^4 \cdot 2du = 2\int u^4\,du = \frac{2}{5}u^5 + C$

let $u = 1 + \frac{t}{2}$

$du = \frac{1}{2}du$

$2du = dt$

$= \frac{2}{5}(1 + \frac{t}{2})^5 + C$

10. $\int (1+2t)^{-4}\,dt = \int u^{-4}\frac{1}{2}du = \frac{1}{2}\int u^{-4}\,du = \frac{1}{2}\frac{u^{-3}}{-3} + C = \frac{-1}{6}(1+2t)^{-3} + C$

$u = 1+2t$

$du = 2dt \rightarrow \frac{1}{2}du = dt$

12. $\int \frac{1}{1+4t}\,dt = \int \frac{1}{u}\cdot\frac{1}{4}\,du = \frac{1}{4}\ln|u| + C = \frac{1}{4}\ln|1+4t| + C$

$u = 1+4t$

$du = 4\,dt$

$\frac{1}{4}du = dt$

14. $\int e^t (1+e^t)^4 dt = \int u^4 du = \frac{1}{5}u^5 + C$

$= \frac{1}{5}(1+e^t) + C$

let $u = 1+e^t$

$du = e^t dt$

20. $\int \frac{\ln x}{x} dx = \int u\, du = \frac{1}{2}u^2 + C$

$= \frac{1}{2}(\ln x)^2 + C$

let $u = \ln x$

$du = \frac{1}{x}dx$

22. $\int x\cos(3x)\, dx = x\left(\frac{1}{3}\sin(3x)\right) - \int \frac{1}{3}\sin(3x)dx = \frac{1}{3}x\sin(3x) - \frac{1}{3}\int \sin(3x)dx$

let $u = x$ $\quad dv = \cos(3x)dx$ $\quad = \frac{1}{3}x\sin(3x) - \frac{1}{3}\left[-\frac{1}{3}\cos(3x)\right] + C$

$du = dx$ $\quad v = \frac{1}{3}\sin(3x)$ $\quad \boxed{= \frac{1}{3}x\sin(3x) + \frac{1}{9}\cos(3x) + C}$

24. $\int x^2 e^{-x} dx = x^2(-e^{-x}) - \int(-e^{-x})2x\, dx = -x^2e^{-x} + 2\int xe^{-x}dx$

1) let $u = x^2$ $\quad dv = e^{-x}dx$ $\quad = -x^2e^{-x} + 2\left[-xe^{-x} - \int -e^{-x}dx\right] = -x^2e^{-x} - 2xe^{-x} + 2\int e^{-x}dx$

$du = 2x dx$ $\quad v = -e^{-x}$ $\quad \boxed{= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C}$

2) $u = x$ $\quad dv = e^{-x}$

$du = dx$ $\quad v = -e^{-x}$

32. $\frac{dP}{dt} = 5e^{-0.2t}$ $\qquad P(0) = 0$

$\int 5e^{-0.2t}dt = \int 5e^u \frac{-1}{0.2}du = \frac{-5}{0.2}\int e^u du = \frac{-5}{0.2}e^u + C$

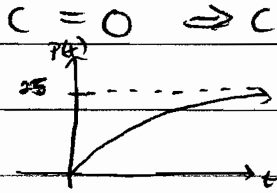
let $u = -0.2t$ $\qquad P(t) = \frac{-5}{0.2}e^{-0.2t} + C$

$du = -0.2\, dt$

$\frac{-1}{0.2}du = dt$ $\qquad P(0) = \frac{-5}{0.2}e^{-0.2(0)} + C = 0$

$-25e^0 + C = 0 \Rightarrow C = 25$

$\boxed{P(t) = -25e^{-0.2t} + 25}$



$\lim_{t\to\infty} P(t) = 25$

$P(10) = -25e^{-0.2(10)} + 25 = \boxed{21.617}$

$P(100) = -25e^{-0.2(100)} + 25 = \boxed{25}$

P(t) does not increase to infinity because its production rate, $\frac{dP}{dt}$ decreases to zero.

36. $\frac{dw}{dt} = (4t - t^2)e^{-3t}$ $\quad w(0) = 0$

Let $f(t) = (4t - t^2)e^{-3t}$. The worm grows fastest when $f'(t) = 0$

$f'(t) = (4t - t^2)\cdot -3e^{-3t} + e^{-3t}(4 - 2t) = e^{-3t}((4t - t^2)(-3) + (4 - 2t))$

$= e^{-3t}(-12t + 3t^2 + 4 - 2t) = e^{-3t}(3t^2 - 14t + 4) = 0$

$f'(t) = 0$ when $3t^2 - 14t + 4 = 0$ $\qquad t = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(3)(4)}}{2(3)} = \frac{14 \pm 12.17}{6}$

$t = 4.36 \approx 0.30$

$f(0.306) = 0.451 \qquad f(4.36) \approx -3.28 \times 10^{-6} \approx 0$

Thus, the worm grows fastest at $t = 0.306$ and the max. growth rate is 0.451

Find $W(2)$ (Solve the diff. eq)

$\int (4t - t^2)e^{-3t}\,dt = (4t - t^2)(-\frac{1}{3}e^{-3t}) - \int \frac{-1}{3}e^{-3t}(4 - 2t)\,dt$

1) $u = 4t - t^2 \quad dv = e^{-3t}dt \qquad = \frac{-1}{3}e^{-3t}(4t - t^2) + \frac{1}{3}\int (4 - 2t)e^{-3t}\,dt$

$du = (4 - 2t)dt \quad v = \frac{-1}{3}e^{-3t} \qquad = \frac{-1}{3}e^{-3t}(4t - t^2) + \frac{1}{3}\left[(4 - 2t)(-\frac{1}{3}e^{-3t}) - \int \frac{-1}{3}e^{-3t}(-2dt)\right]$

2) $u = 4 - 2t \quad dv = e^{-3t}dt \qquad = \frac{-1}{3}e^{-3t}(4t - t^2) - \frac{1}{9}e^{-3t}(4 - 2t) - \frac{2}{9}\int e^{-3t}\,dt$

$du = -2dt \quad v = \frac{-1}{3}e^{-3t} \qquad = \frac{-1}{3}e^{-3t}(4t - t^2) - \frac{1}{9}e^{-3t}(4 - 2t) - \frac{2}{9}\cdot\frac{-1}{3}e^{-3t} + C$

$W(t) = \frac{-1}{3}e^{-3t}(4t - t^2) - \frac{1}{9}e^{-3t}(4 - 2t) + \frac{2}{27}e^{-3t} + C$

$w(0) = 0 \qquad w(0) = \frac{-1}{3}e^{-3(0)}(4(0) - 0^2) - \frac{1}{9}e^{-3(0)}(4 - 2(0)) + \frac{2}{27}e^{-3(0)} + C = 0$

$0 - \frac{1}{9}(4) + \frac{2}{27} + C = 0 \qquad C = \frac{4}{9} - \frac{2}{27} = \frac{10}{27}$

$w(t) = \frac{-1}{3}e^{-3t}(4t - t^2) - \frac{1}{9}e^{-3t}(4 - 2t) + \frac{2}{27}e^{-3t} + \frac{10}{27}$

$w(2) = \frac{-1}{3}e^{-3(2)}(4(2) - 2^2) - \frac{1}{9}e^{-3(2)}(4 - 2(2)) + \frac{2}{27}e^{-3(2)} + \frac{10}{27} \approx 0.367$

If the worm always grew at a rate of 0.451 at time 2 it would be $2(0.451) = 0.902$ and would be $0.902 - 0.367 = 0.535$ units longer.

$\frac{dL}{dt} = \alpha e^{-\beta t}$ $\alpha = 6.48$ $\beta = 0.06$

a) $\frac{dL}{dt} = 6.48 e^{-0.06t}$

$\int 6.48 e^{-0.06t} dt = 6.48 \int e^{u} \frac{1}{-0.06} du = \frac{6.48}{-0.06} e^{-0.06t} + C = -108 e^{-0.06t} + C = L(t)$

let $u = -0.06t$

$du = -0.06\, dt$

$\frac{1}{-0.06} du = dt$

$L(0) = -108 e^{-0.06(0)} + C = 0 \Rightarrow C = 108$

$L(t) = -108 e^{-0.06t} + 108$

b) $\lim_{t \to \infty} -108 e^{-0.06t} + 108 = 108$

c) $45 = -108 e^{-0.06t} + 108$

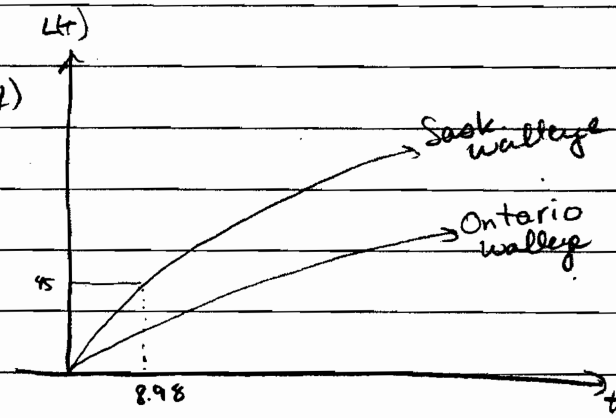
$-63 = -108 e^{-0.06t}$

$0.58\overline{3} = e^{-0.06t}$

$\ln(0.583) = -0.06t$

$t = 8.98$ years

d)



The Saskatchewan walleyes reach maturity earlier and grow lager that the Ontario walleye.

58. Saskatchewan Walleye

$\frac{dL}{dt} = \alpha e^{-\beta t}$ $\quad \alpha = 6.48 \quad \beta = 0.06$

a) $\frac{dL}{dt} = 6.48 e^{-0.06t}$

$\int 6.48 e^{-0.06} dt = 6.48 \int e^{u} \frac{1}{-0.06} du = \frac{6.48}{-0.06} e^{-0.06t} + C = -108 e^{-0.06t} + C = L(t)$

let $u = -0.06t$

$du = -0.06\, dt$

$\frac{1}{-0.06} du = dt$

$L(0) = -108 e^{-0.06(0)} + C = 0 \Rightarrow C = 108$

$L(t) = -108 e^{-0.06t} + 108$

b) $\lim_{t \to \infty} -108 e^{-0.06t} + 108 = 108$

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$\ln(0.583) = -0.06t$

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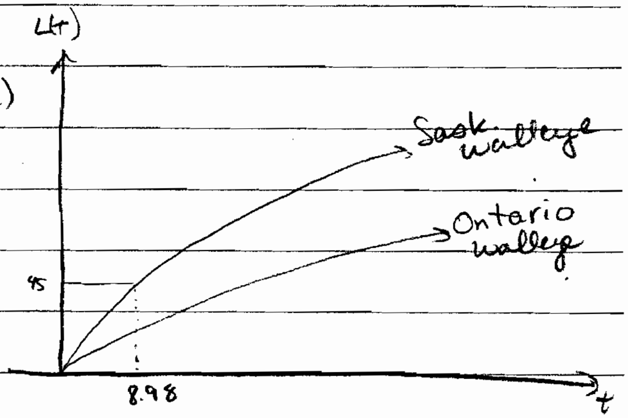

The Saskatchewan walleyes reach maturity earlier and grow larger that the Ontario walleye.