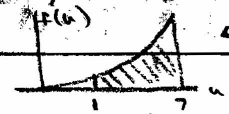


← new graph

4. $f(t)=(1+3t)^3 \quad t=0 \text{ to } t=2$

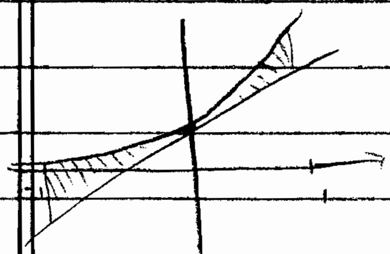
$\int_0^2 (1+3t)^3\,dt = \int_1^7 u^3 \tfrac{1}{3}du = \tfrac{1}{3}\int_1^7 u^3 du = \tfrac{1}{3}\left.\tfrac{u^4}{4}\right]_1^7 = \tfrac{1}{12}\left[7^4-1^4\right] = \tfrac{2400}{12} = 200$

$u=1+3t \qquad u(0)=1+3(0)=1$
$\qquad\qquad\quad u(2)=1+3(2)=7$

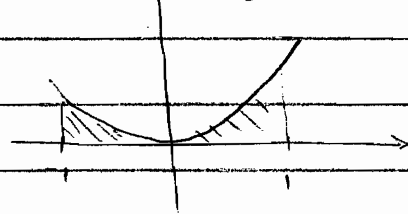
$\frac{du}{dt}=3 \Rightarrow \tfrac{1}{3}du=dt$

8. $f(x)=e^x \quad g(x)=x+1 \quad -1\le x\le 1$

a)



b) $f(x)-g(x)=e^x-x-1$

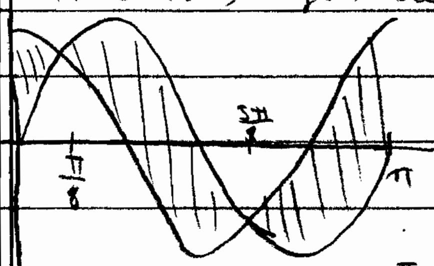


c) $A=\int_{-1}^{1}\left(e^x-(x+1)\right)dx = \int_{-1}^{1}\left(e^x-x-1\right)dx = \left.e^x-\tfrac{1}{2}x^2-x\right]_{-1}^{1}$

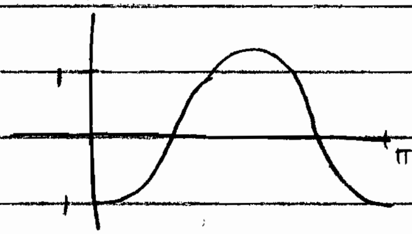
$= e^1-\tfrac{1}{2}(1)^2-1-\left(e^{-1}-\tfrac{1}{2}(-1)^2-(-1)\right)\approx 0.35$

12 $f(x)=\sin(2x) \quad g(x)=\cos(2x) \quad 0\le x\le\pi$

a)



b)



side:
$\int\cos(2x)dx$
$=\tfrac{1}{2}\sin(2x)+C$
$\int\sin(2x)\,dx$
$=\tfrac{-1}{2}\cos(2x)+C$

$A=A_1+A_2+A_3 = \int_0^{\pi/8}\cos(2x)-\sin(2x)\,dx + \int_{\pi/8}^{5\pi/8}\left(\sin(2x)-\cos(2x)\right)dx + \int_{5\pi/8}^{\pi}\left(\cos(2x)-\sin(2x)\right)dx$

or notice that $A_1+A_3=A_2$. So, $A=2A_2$

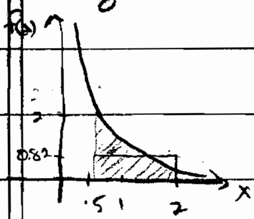
$A=2\int_{\pi/8}^{5\pi/8}\left(\sin(2x)-\cos(2x)\right)dx = 2\left[-\tfrac{1}{2}\cos(2x)-\tfrac{1}{2}\sin(2x)\right]_{\pi/8}^{5\pi/8}$

$=2\left[\tfrac{-1}{2}\cos\left(\tfrac{10}{8}\pi\right)-\tfrac{1}{2}\sin\left(\tfrac{10\pi}{8}\right)-\left(-\tfrac{1}{2}\cos\left(\tfrac{2\pi}{8}\right)-\tfrac{1}{2}\sin\left(\tfrac{2\pi}{8}\right)\right)\right]$

$=2\left[\tfrac{-1}{2}\left(\tfrac{\sqrt2}{2}\right)-\tfrac{1}{2}\left(\tfrac{-\sqrt2}{2}\right)-\left(\tfrac{-1}{2}\left(\tfrac{-\sqrt2}{2}\right)-\tfrac{1}{2}\left(\tfrac{\sqrt2}{2}\right)\right)\right]=\sqrt2+\sqrt2=2\sqrt2\approx 2.828$

14. $f(x)=\frac{1}{x}$ $\quad 0.5\le x\le 2$

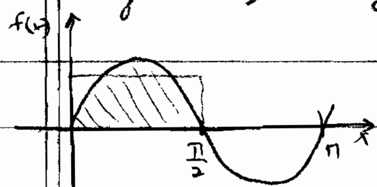
$$f_{avg} = \frac{1}{2-0.5}\int_{0.5}^{2}\frac{1}{x}\,dx = \frac{1}{1.5}\ln|x|\Big]_{.5}^{2} = \frac{2}{3}\left[\ln|2|-\ln|0.5|\right] = 0.92$$



16. $f(x)=\sin(2x)$ $\quad 0\le x\le \frac{\pi}{2}$

$$f_{avg} = \frac{1}{\frac{\pi}{2}-0}\int_{0}^{\frac{\pi}{2}}\sin(2x)\,dx = \frac{2}{\pi}\left(-\frac{1}{2}\cos(2x)\right]_{0}^{\frac{\pi}{2}} = -\frac{1}{\pi}\cos(2x)\Big]_{0}^{\frac{\pi}{2}}$$

$$= -\frac{1}{\pi}\left[\cos(\pi)-\cos(0)\right] = -\frac{1}{\pi}\left[-1-1\right] = -\frac{1}{\pi}\left[-2\right] = \frac{2}{\pi}\approx 0.63$$

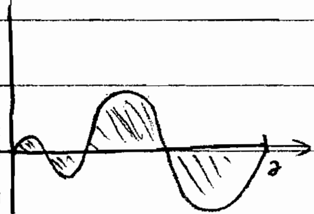


18. $$A = \int_{0}^{2} x\sin(2\pi x)\,dx = x\left(\frac{-1}{2\pi}(\cos(2\pi x))\right)\Big]_{0}^{2} - \int_{0}^{2}\frac{-1}{2\pi}\cos(2\pi x)\,dx$$

$u = x$ $\quad dv = \sin(2\pi x)\,dx$ $\quad = \frac{-x}{2\pi}\cos(2\pi x)\Big]_{0}^{2} + \frac{1}{2\pi}\int_{0}^{2}\cos(2\pi x)\,dx$

$du = dx$ $\quad v = \frac{-1}{2\pi}\cos(2\pi x)$ $\quad = \frac{-x}{2\pi}\cos(2\pi x)\Big]_{0}^{2} + \frac{1}{2\pi}\left[\frac{1}{2\pi}\sin(2\pi x)\right]_{0}^{2}$

$$= \frac{-2}{2\pi}\cos(4\pi) + \frac{1}{(2\pi)^2}\sin(4\pi) - \left(0 + \frac{1}{(2\pi)^2}\sin(0)\right) = -\frac{1}{\pi}(1) + \frac{1}{4\pi^2}(0) \approx -0.318$$



Yes, the answer makes since because there is a greater area below the x-axis

30. $g(t) = 360t - 39t^2 + t^3$

$$\int_{15}^{24}(360t-39t^2+t^3)\,dt = \frac{360}{2}t^2 - \frac{39}{3}t^3 + \frac{1}{4}t^4\Big]_{15}^{24} = 180t^2 - 13t^3 + \frac{1}{4}t^4\Big]_{15}^{24}$$

$$= 6912 - 9281.25 = \boxed{-2369.25}$$

$$g_{avg} = \frac{1}{24-15}\int_{15}^{24}(360t-39t^2+t^3)\,dt = \frac{1}{9}(-2369.25) = \boxed{-263.25}$$

34) $\rho(x) = 1 + 2\times10^{-8}x^2(240-x) = 1 + 4.8\times10^{-6}x^2 - 2\times10^{-8}x^3$

a) $\rho'(x) = 9.6\times10^{-6}x - 6\times10^{-8}x^2 = 0$

$x(9.6\times10^{-6} - 6\times10^{-8}x) = 0 \qquad x=0 \qquad 9.6\times10^{-6} - 6\times10^{-8}x = 0$

$x = \frac{9.6\times10^{-6}}{6\times10^{-8}} = 160$

$\rho(0) = 1$ minimum $\qquad \rho(160) \approx 1.0491$ maximum

b) $\int_0^{200} (1 + 4.8\times10^{-6}x^2 - 2\times10^{-8}x^3)\,dx = x + \frac{4.8\times10^{-6}}{3}x^3 - \frac{2\times10^{-8}}{4}x^4 \Big]_0^{200}$

$= 200 + \frac{4.8\times10^{-6}}{3}(200)^3 - \frac{2\times10^{-8}}{4}(200)^4 - 0 = \boxed{204.8\ g}$

c) $\rho_{avg} = \frac{1}{200-0}\int_0^{200}\rho(x)\,dx = \frac{1}{200}(204.8) = \boxed{1.024\ \frac{g}{m}}$

d)