math 1210 HW 34 Sec 4.7: 6, 8, 10, 13, 14, 18, 20, 22, 28

6. $\int_0^\infty e^t\,dt = \lim_{T\to\infty}\int_0^T e^t\,dt = \lim_{T\to\infty} e^t\Big]_0^T = \lim_{T\to\infty} e^T - e^0 = \infty$ $\boxed{\text{Divergent}}$

8. $\int_5^\infty \frac{1}{x^2}\,dx = \lim_{t\to\infty}\int_5^t x^{-2}\,dx = \lim_{t\to\infty} -x^{-1}\Big]_5^t = \lim_{t\to\infty} \frac{-1}{t} - \frac{-1}{5} = \boxed{\frac{1}{5}}$ $\boxed{\text{convergent}}$

10. $\int_0^\infty \frac{1}{(2+5x)^4}\,dx = \int_2^\infty \frac{1}{u^4}\frac{1}{5}\,du = \frac{1}{5}\int_2^\infty u^{-4}\,du = \lim_{t\to\infty}\frac{1}{5}\int_2^t u^{-4}\,du = \lim_{t\to\infty}\frac{1}{5}\,\frac{-1}{3}\,u^{-3}\Big]_2^t$

$u = 2+5x$

$du = 5\,dx$

$\frac{1}{5}du = dx$

$u(0) = 2+5(0) = 2$

$= \lim_{t\to\infty}\frac{-1}{15}\left[\frac{1}{t^3} - \frac{1}{2^3}\right] = \frac{1}{120}$ $\boxed{\text{convergent}}$

12. $\int_0^{0.001} \frac{1}{x^2}\,dx = \lim_{t\to 0^+}\int_t^{0.001} x^{-2}\,dx = \lim_{t\to 0^+} -x^{-1}\Big]_t^{0.001} = \lim_{t\to 0^+}\frac{-1}{0.001} - \frac{-1}{t} = \infty$ $\boxed{\text{divergent}}$

13. $\int_0^{0.001} \frac{1}{\sqrt[3]{x}}\,dx = \lim_{t\to 0^+}\int_t^{0.001} x^{-\frac{1}{3}}\,dx = \lim_{t\to 0^+}\frac{3}{2}x^{\frac{2}{3}}\Big]_t^{0.001} = \lim_{t\to 0^+}\frac{3}{2}(0.001)^{\frac{2}{3}} - \frac{3}{2}(t)^{\frac{2}{3}} = 0.015$ $\boxed{\text{convergent}}$

14. $\int_0^\infty \frac{1}{\sqrt[3]{x}}\,dx = \int_0^1 \frac{1}{\sqrt[3]{x}}\,dx + \int_1^\infty \frac{1}{\sqrt[3]{x}}\,dx = \lim_{t\to 0^+}\int_t^1 x^{-\frac{1}{3}}\,dx + \lim_{t\to\infty}\int_1^t x^{-\frac{1}{3}}\,dx$

$= \lim_{t\to 0^+}\frac{3}{2}x^{\frac{2}{3}}\Big]_t^1 + \lim_{t\to\infty}\frac{3}{2}x^{\frac{2}{3}}\Big]_1^t$

$= \lim_{t\to 0^+}\frac{3}{2}(1)^{\frac{2}{3}} - \frac{3}{2}(t)^{\frac{2}{3}} + \lim_{t\to\infty}\frac{3}{2}(t)^{\frac{2}{3}} - \frac{3}{2}(1)^{\frac{2}{3}} = 1.5 - 0 + \infty - 1.5 = \infty$

$\therefore \int_0^\infty \frac{1}{\sqrt[3]{x}}\,dx$ diverges

18. $\int \frac{1}{\sqrt{x}+e^x} dx$

The leading behavior as $x \to \infty$ is e^x

The integral of leading behavior is $\int_1^{\infty} \frac{1}{e^x} dx = \int_1^{\infty} e^{-x} dx$

$\int_1^{\infty} e^{-x} dx = \lim_{t\to\infty} \int_1^t e^{-x} dx = \lim_{t\to\infty} -e^{-x}\Big]_1^t = \lim_{t\to\infty} -e^{-t} + e^{-1} = e^{-1} = \frac{1}{e} \approx 0.37$

So, then $\int_1^{\infty} \frac{1}{\sqrt{x}+e^x} dx$ converges.

20 $\int_0^1 \frac{1}{x^2+e^x} dx$ ~~notice:~~ $\frac{1}{x^2+e^x} \leq \frac{1}{e^x}$

*not improper ~~$\int_0^1 \frac{1}{e^x} dx = \int_0^1 e^{-x} dx = -e^{-x}\Big]_0^1 = -e^{-1} - -e^0 = \frac{-1}{e} + 1 \approx 0.632$~~

$\therefore \int_0^1 \frac{1}{x^2+e^x} dx$ converges to a value < 0.632.

22. $\int_1^{\infty} \frac{1}{\sqrt{x}+e^x} dx$ notice: $\frac{1}{\sqrt{x}+e^x} \leq \frac{1}{e^x}$

from 18 $\int_1^{\infty} \frac{1}{e^x} dx = \int_1^{\infty} e^{-x} dx = \frac{1}{e} \approx 0.37$

$\therefore \int_1^{\infty} \frac{1}{\sqrt{x}+e^x} dx$ converges and converges to a value < 0.37

28 $\frac{dC}{dt} = 50e^{-2t}$ $C(0)=10$ Let K be the constant of integration here

$C(t) = \int 50e^{-2t} dt = \frac{50}{-2} e^{-2t} + K$

$C(0) = -25e^{-2(0)} + K = 10 \Rightarrow K = 35$ $C(t) = -25e^{-2t} + 35$

$\lim_{t\to\infty} C(t) = \lim_{t\to\infty} -25e^{-2t} + 35 = 35$ No, this cell would not survive.

or $C(T) = 10 + \int_0^T 50e^{-2t} dt = 10 + \frac{50}{-2} e^{-2t}\Big]_0^T = 10 + -25e^{-2T} - -25e^{-2(0)}$

$= 35 - 25e^{-2T}$

$\lim_{T\to\infty} C(T) = \lim_{T\to\infty} 35 - 25e^{-2T} = 35$

exceeding a concentration of 30 $\frac{\mu mol}{L}$

and the cell dies.

2. $\frac{dx}{dt} = \frac{x^2}{x-2}$ autonomous; state variable: x

3. $\frac{dy}{dt} = \mu e^{-t} - 1$ pure time; state variable: y

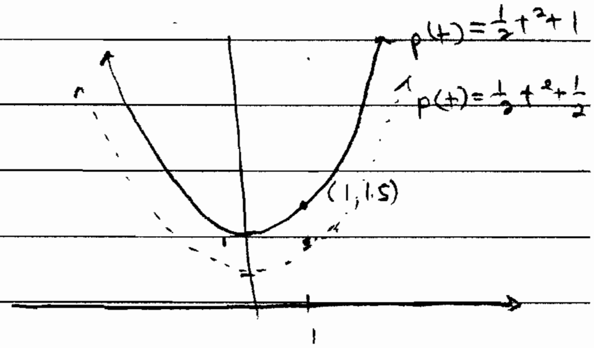
4. $\frac{dm}{dt} = \frac{e^{\cancel{\alpha m}} m^2}{mt - \lambda}$ nonautonomous (mixed); state variable: m

6. $t = 0 \quad x = 1, \lambda = 2$

$\frac{dx}{dt} = \frac{(1)^2}{1-2} = \frac{1}{-1} = -1$ The state variable, x is decreasing

8. $t = 2 \quad m = 0 \quad \alpha = 2 \quad \lambda = 1$ The state variable, m is remaining unchanged

$\frac{dm}{dt} = \frac{e^{2(10)}(0)^2}{(0.2-1)} = \frac{0}{-1} = 0$



9. $\frac{dP}{dt} = t \quad p(0) = 1$

$p(t) = \frac{1}{2}t^2 + C$

$p(0) = \frac{1}{2}(0)^2 + C = 1 \Rightarrow C = 1$

$p(t) = \frac{1}{2}t^2 + 1$

$p(1) = \frac{1}{2}(1)^2 + 1 = 1.5$

10. now $p(1) = 1$

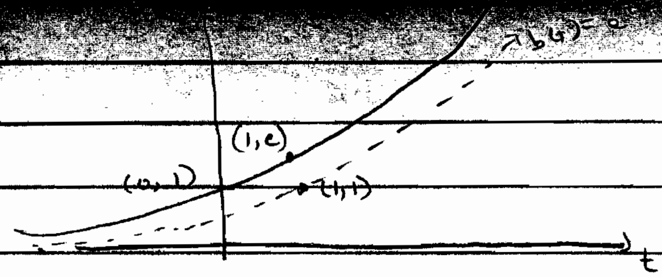
$p(1) = \frac{1}{2}(1)^2 + C = 1 \Rightarrow C = \frac{1}{2}$

$p(t) = \frac{1}{2}t^2 + \frac{1}{2}$

$p(2) = \frac{1}{2}(2)^2 + \frac{1}{2} = 2.5$

(see dotted line on above graph.)

[illegible] $b(0)=1$ $b(t)=e^{t}$



LHS: $\frac{db}{dt} = b'(t) = e^t$ ✓

RHS: $b = e^t$ ✓

Initial condition $b(0)=e^0=1$

$b(1)=e^1=e$

12. $b(1)=1$ $b(t)=e^{t-1}$

LHS: $\frac{db}{dt} = b'(t) = e^{t-1}(1) = e^{t-1}$ ✓

RHS: $b = e^{t-1}$ ✓ (see dotted line on above graph)

Initial condition: $b(1) = e^{1-1} = e^0 = 1$

$b(2) = e^{2-1} = e^1 = e$

14. They match because for an autonomous differential equation the change is not dependent on time, but only on the size of the state variable.

16. $b(t) = 10e^{3t}$ $\frac{db}{dt} = 3b$ $b(0)=10$

LHS: $\frac{db}{dt} = b'(t) = 10e^{3t}(3) = 30e^{3t}$ ✓

RHS: $3b = 3(10e^{3t}) = 30e^{3t}$ ✓

Initial condition: $b(0) = 10e^{3(0)} = 10$ ✓

18. $z(t) = 1+\sqrt{1+2t} = 1+(1+2t)^{\frac{1}{2}}$ $\frac{dz}{dt} = \frac{1}{z-1}$ $z(0)=2$

LHS: $\frac{dz}{dt} = z'(t) = \frac{1}{2}(1+2t)^{-\frac{1}{2}}(2) = \frac{1}{\sqrt{1+2t}}$ ✓

RHS: $\frac{1}{z-1} = \frac{1}{1+(1+2t)^{\frac{1}{2}}-1} = \frac{1}{(1+2t)^{\frac{1}{2}}} = \frac{1}{\sqrt{1+2t}}$ ✓

Initial condition: $z(0) = 1+\sqrt{1+2(0)} = 1+\sqrt{1} = 1+1 = 2$ ✓