Math 1290 HW 36 Sec 5.# : 20, 22, 28, 29, 32, 33, 34, 40, 42, 44

20. $\frac{db}{dt} = 3b \quad b(0)=10 \quad \Delta t = 0.5$

$\hat{b}(0.5) = b(0) + b'(0)\Delta t = 10 + 3(10)(0.5) = 25$

$\hat{b}(1) = \hat{b}(0.5) + b'(0.5)\Delta t = 25 + 3(25)\,0.5 = 62.5$

Exact answer from #16 $b(t) = 10e^{3t}$ $\quad b(1) = 10e^{3(1)} \approx 200.85$

Euler's method provides a very low estimate.

22. $\frac{dz}{dt} = \frac{1}{z-1} \quad z(0) = 2 \quad \Delta t = 1$

$\hat{z}(1) = z(0) + z'(0)\Delta t = 2 + \left(\frac{1}{2-1}\right)1 = 3$

$\hat{z}(2) = \hat{z}(1) + z'(1)\Delta t = 3 + \left(\frac{1}{3-1}\right)1 = 3.5$

$\hat{z}(3) = \hat{z}(2) + z'(2)\Delta t = 3.5 + \left(\frac{1}{3.5-1}\right)(1) = 3.9$

$\hat{z}(4) = \hat{z}(3) + z'(3)\Delta t = 3.9 + \left(\frac{1}{3.9-1}\right)(1) = \boxed{4.245}$

Exact answer from #18 : $z(t) = 1 + \sqrt{1+2t}$

$z(4) = 1 + \sqrt{1+2(4)} = \boxed{4}$ Here Euler's method is close.

28. $\frac{db}{dt} = \lambda(b)\cdot b \quad \lambda(0) = 4 \quad$ slope $m = -0.001 \quad \lambda(b) = -0.001b + 4$

$\frac{db}{dt} = (-0.001b+4)b = -0.001b^2 + 4b$

$b = 1000 \quad \frac{db}{dt} = -0.001(1000)^2 + 4(1000) = 3000$ The population, b(t) is increasing

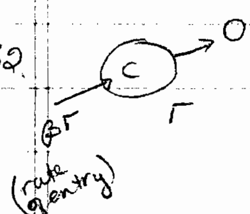
$b = 5000 \quad \frac{db}{dt} = -0.001(5000)^2 + 4(5000) = -5000.$ The population, b(t) is decreasing

29. $\lambda(0) = -2 \quad$ slope $m = 0.01 \quad \lambda(b) = 0.01b - 2$

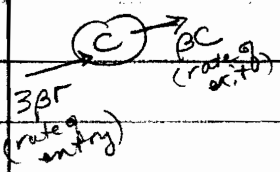
$\frac{db}{dt} = (0.01b - 2)b = 0.01b^2 - 2b$

$b = 100 \quad \frac{db}{dt} = 0.01(100)^2 - 2(100) = -200$ The population, b(t) is decreasing

$b = 300 \quad \frac{db}{dt} = 0.01(300)^2 - 2(300) = 300$ The population, b(t) is increasing

32. 

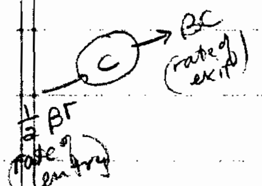
$\frac{dC}{dt} = \beta\Gamma$ The concentration increases

33. 

$\frac{dC}{dt} = 3\beta\Gamma - \beta C$

If $C = \Gamma$ $\quad \frac{dC}{dt} = 3\beta C - \beta C = 2\beta C > 0$

So, the concentration in the cell would be increasing and would become larger than the concentration outside the cell.

34. 

$\frac{dC}{dt} = \frac{1}{2}\beta\Gamma - \beta C$

If $C = \Gamma$ then $\frac{dC}{dt} = \frac{1}{2}\beta C - \beta C = -\frac{1}{2}\beta C < 0$

So, the concentration in the cell is decreasing and will become lower than the concentration outside

40. $\frac{dH}{dt} = \alpha(A - H) \qquad H(t) = A + (H(0) - A)e^{-\alpha t}$

a) $H(t) = 30 + (40-30)e^{-0.02t} = 30 + 10e^{-0.02t}$

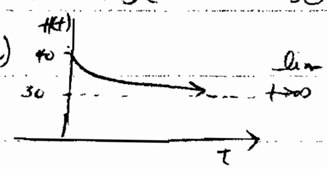
LHS: $\frac{dH}{dt} = H'(t) = 10e^{-0.02t}(-0.02) = -0.2e^{-0.02t}$ ✓

RHS: $\alpha(A-H) = 0.02\left(30 - (30 + 10e^{-0.02t})\right) = 0.02\left(-10e^{-0.02t}\right) = -0.2e^{-0.02t}$ ✓

Initial condition $H(0) = 40 \qquad H(0) = 30 + 10e^{-0.02(0)} = 30 + 10 = 40$ ✓

b) $H(1) = 30 + 10e^{-0.02(1)} = 39.80°C$

$H(2) = 30 + 10e^{-0.02(2)} = 39.61°C$

c) 

$\lim_{t\to\infty} H(t) = \lim_{t\to\infty} 30 + 10e^{-0.02t} = \boxed{30}$

42. $\alpha = 0.02 \quad A = 30 \quad H(0) = 40 \quad \Delta t = 1 \qquad \frac{dH}{dt} = 0.02(30 - H(t))$

$H(1) = H(0) + H'(0)\Delta t = 40 + 0.02(30-40)1 = 39.8°C$

$H(2) = H(1) + H'(1)\Delta t = 39.8 + 0.02(30 - 39.8)(1) = 39.604°C$

Euler's method is very close because the temperature change is linear

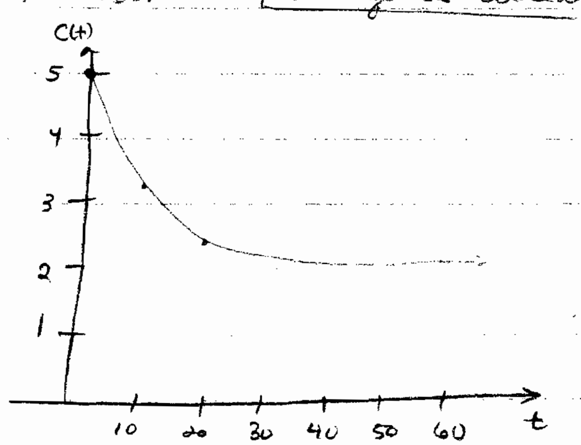
44. $C(t) = \Gamma + (C(0) - \Gamma)e^{-\beta t}$

$C(t) = 2 + (5-2)e^{-0.01t} = 2 + 3e^{-0.1t}$

$C(10) = 2 + 3e^{-0.1(10)} = 3.10$

$C(20) = 2 + 3e^{-0.1(20)} = 2.41$

$C(60) = 2 + 3e^{-0.1(60)} = 2.01$



Math 1290 HW 37 Sec 5.2: 2, 4, 6, 7, 10, 12, 14, 18, 24, 26, 29, 30

2. $\frac{dx}{dt} = 1 - e^x$

$\frac{dx}{dt} = 0$ when $1 - e^x = 0$ $\quad 1 = e^x \to \ln(1) = \ln(e^x)$ $\quad \boxed{\text{eq pt } x = \ln(1) = 0}$

4) $\frac{dz}{dt} = \frac{1}{z} - 3$

$\frac{dz}{dt} = 0$ when $\frac{1}{z} - 3 = 0$ $\quad \frac{1}{z} = 3$ $\quad \boxed{\text{eq. pt: } z = \frac{1}{3}}$

6) $\frac{dx}{dt} = cx + x^2$

$\frac{dx}{dt} = 0$ when $cx + x^2 = 0$

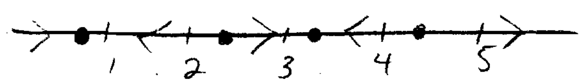
$x(c + x) = 0$ $\quad \boxed{\text{eq pts: } x = 0 \quad x = -c}$

7. $\frac{dw}{dt} = \alpha e^{\beta w} - 1$

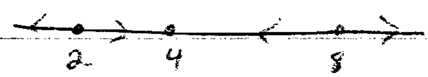
$\frac{dw}{dt} = 0$ when $\alpha e^{\beta w} - 1 = 0$ $\quad \alpha e^{\beta w} = 1$ $\quad e^{\beta w} = \frac{1}{\alpha}$ $\quad \beta w = \ln(\frac{1}{\alpha})$

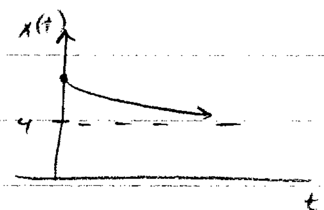
$$w = \frac{\ln(\frac{1}{\alpha})}{\beta} = \boxed{\frac{-\ln(\alpha)}{\beta}}$$

10.

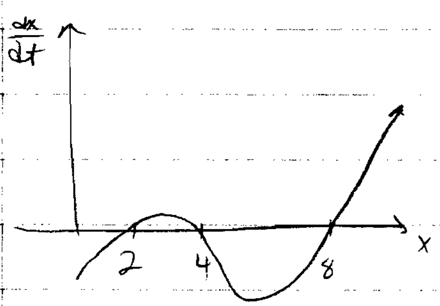


12. Given

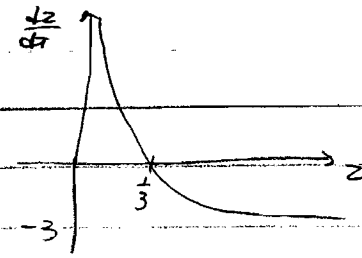




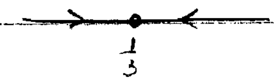
14.



18. $\frac{dz}{dt} = \frac{1}{z} - 3$



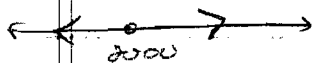
Phase line diagram

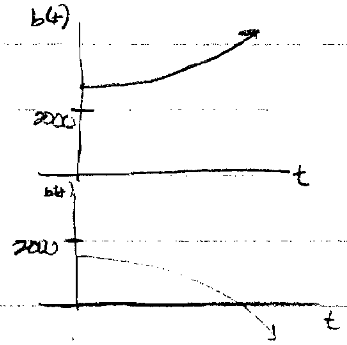


24. $\frac{db}{dt} = \lambda b - h \quad \lambda = 0.5 \quad h = 1000$

$\frac{db}{dt} = 0.5b - 1000 = 0 \qquad 0.5b = 1000$

$b = \frac{1000}{0.5} = \boxed{2000}$



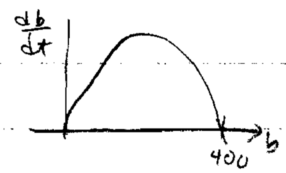


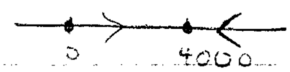
This population can out grow the harvest if it starts at a large enough value (size) (>2000). If the size starts too small (<2000), it will be driven to extinction by the harvest

26. $\frac{db}{dt} = (4 - 0.001b)\,b = 0$

$b = 0 \qquad 4 - 0.001b = 0$

$b = \frac{4}{0.001} = 4000$





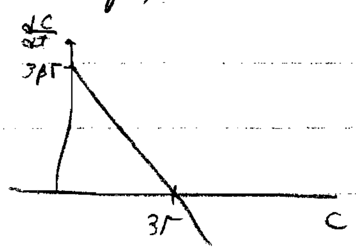
at $b = 1000$ the arrow points right, consistent with an increasing population

at $b = 5000$ the arrow points to the left, consistent with a decreasing population

29. $\frac{dC}{dt} = 3\beta\Gamma - \beta C = 0$

$3\beta\Gamma = \beta C$

$C = 3\Gamma$

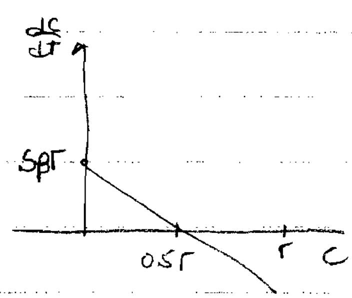


When $C = \Gamma$, $\frac{dC}{dt} > 0$. This is consistent with an increasing internal condition

30. $\frac{dC}{dt} = 0.5\beta\Gamma - \beta C = 0$

$0.5\beta\Gamma = \beta C$

$C = 0.5\Gamma$



When $C = \Gamma$, $\frac{dC}{dt} < 0$. This is consistent with a decreasing internal concentration.