$\frac{db}{dt} = -3b \quad b(0) = 1\times10^6$

$\int \frac{db}{b} = \int -3\,dt \qquad \ln|b| = -3t + C$

$b = e^{-3t+C} = Ce^{-3t}$

$b(0) = Ce^{-3(0)} = 1\times10^6 \Rightarrow C = 1\times10^6$

$b(t) = (1\times10^6)\,e^{-3t}$

Checking: $b'(t) = (1\times10^6)\,e^{-3t}(-3) = -3\,b(t) = \frac{db}{dt}$

4. $\frac{dN}{dt} = 4 + 2N \qquad N(0) = 1$

side: ~~let $u = 4+2N$~~

$\frac{du}{dN} = 2$

$\frac{1}{2}du = dN$

$\int \frac{dN}{4+2N} = \int dt$

$\int \frac{1}{u}\,\frac{1}{2}\,du$

$-\frac{1}{2}\ln|4+2N| = t + C$

$\ln|4+2N| = 2t + C$

$4 + 2N = e^{2t+C} = Ce^{2t}$

$2N = Ce^{2t} - 4$

$N = \frac{Ce^{2t}-4}{2} = Ce^{2t} - 2$

$N(0) = Ce^{2(0)} - 2 = 1$

$C - 2 = 1 \Rightarrow C = 3$

$\boxed{N(t) = 3e^{2t} - 2}$

checking

LHS: $\frac{dN}{dt} = N'(t) = 3e^{2t}\cdot 2 = 6e^{2t}$ ✓

RHS: $4 + 2N = 4 + 2(3e^{2t} - 2) = 6e^{2t}$ ✓

5. $\frac{db}{dt} = 1000 - b \qquad b(0) = 500$

$\int \frac{db}{1000-b} = \int dt$

$-\ln|1000-b| = t + C$

$\ln|1000-b| = -t + C$

$1000 - b = e^{-t+C} = Ce^{-t}$

$b(t) = 1000 - Ce^{-t}$

$b(0) = 1000 - Ce^{-0} = 500 \Rightarrow C = 500$

$\boxed{b(t) = 1000 - 500e^{-t}}$

Checking:

LHS: $\frac{db}{dt} = b'(t) = 500e^{-t}$ ✓

RHS: $1000 - b = 1000 - (1000 - 500e^{-t}) = 500e^{-t}$

[redacted]

$\int dL = \int 1000\, e^{0.2t}\, dt$

$L = \frac{1000}{0.2} e^{0.2t} + C = 5000 e^{0.2t} + C$

$L(0) = 5000\, e^{0.2(0)} + C = 1000 \quad C = -4000$

$L(t) = 5000\, e^{0.2t} - 4000$

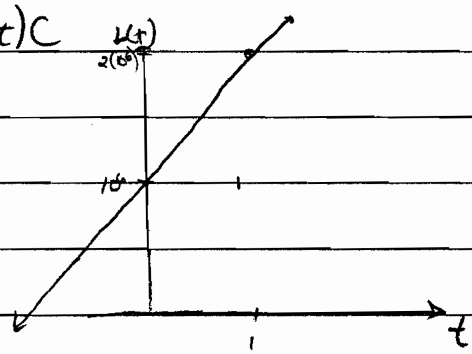
checking: $L'(t) = 5000 e^{0.2t}(0.2) = 1000\, e^{0.2t} = \frac{dL}{dt}$

12. $\frac{db}{dt} = \lambda(t) b$ w/ $\lambda(t) = \frac{1}{1+t}$ $b(0) = 10^6$

$\frac{db}{dt} = \frac{1}{1+t} b$

$\int \frac{db}{b} = \int \frac{1}{1+t}\, dt$

$e^{\ln|b|} = e^{\ln|1+t| + C} = e^{\ln|1+t|} e^C = (1+t)C$



$b = (1+t)C$

$b(0) = (1+0)C = 10^6 \Rightarrow C = 10^6$

$b(t) = 10^6(1+t)$

checking: LHS: $\frac{db}{dt} = b'(t) = 10^6$ ✓

RHS: $\frac{b}{1+t} = \frac{10^6\cancel{(1+t)}}{\cancel{1+t}} = 10^6$ ✓

13. $\frac{db}{dt} = \lambda(t) b$ w/ $\lambda(t) = \cos(t)$ $b(0) = 10^6$

$\frac{db}{dt} = \cos(t)\, b$

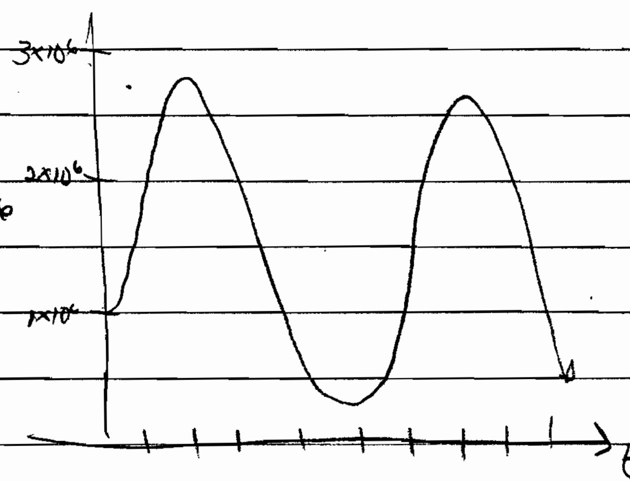
$\int \frac{db}{b} = \int \cos t\, dt$

$\ln|b| = \sin t + C$

$b = e^{\sin t + C} = e^{\sin t} e^C = C e^{\sin t}$

$b(0) = C e^{\sin(0)} = 10^6 \quad C e^0 = 10^6 \quad C = 10^6$

$b(t) = 10^6\, e^{\sin t}$



checking:

LHS: $\frac{db}{dt} = 10^6\, e^{\sin t} \cos t$ ✓

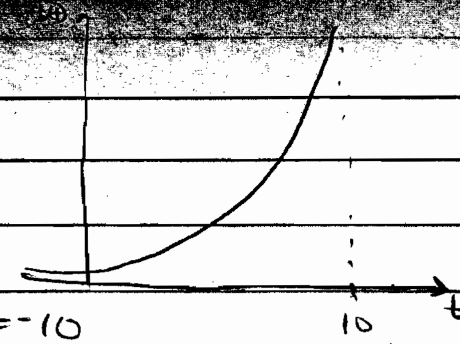
RHS: $\cos(t) \cdot b = \cos(t) \cdot 10^6 e^{\sin t}$ ✓

16. $\frac{db}{dt} = b^p$ w/ $p=2$ $b(0) = 0.1$

$\frac{db}{dt} = b^2$

$\int \frac{db}{b^2} = \int dt$

$-\frac{1}{b} = t + C \Rightarrow b = \frac{-1}{t+C}$



$b(0) = \frac{-1}{0+C} = 0.1$ $\quad \frac{-1}{c} = 0.1 \Rightarrow c = \frac{-1}{0.1} = -10$

$b(t) = \frac{-1}{t-10}$ $\quad$ The solution approaches infinity at $t=10$

23. $\frac{dc}{dt} = \beta(\Gamma - c)$ $\quad \beta = 0.1$ $\quad c(0) = 5$ $\quad \Gamma = 2$

$\frac{dc}{dt} = 0.1(2-c)$

$\int \frac{dc}{2-c} = \int 0.1\, dt$

$-\ln|2-c| = 0.1t + K$

$\ln|2-c| = -0.1 + k$

$2 - c = e^{-0.1t+k} = Ke^{-0.1t}$

$c = 2 - K e^{-0.1t}$

$C(0) = 2 - Ke^{-0.1(0)} = 5$

$2 - K = 5 \Rightarrow k = -3$

$C(t) = 2 - (-3)e^{-0.1t} = \boxed{2 + 3e^{-0.1t}}$

$C(10) = 2 + 3e^{-0.1(10)} = 3.10$

The equilibrium value is $\Gamma = 2$ $\quad C(0) = 5$ $\quad$ halfway is 3.5

$C(t) = 2 + 3e^{-0.1(t)} = 3.5$

$3e^{-0.1t} = 1.5$

$e^{-0.1t} = \frac{1.5}{3}$

$t = 6.93$

$\int \frac{dy}{\sqrt{y}} = \int -2dt$

$\int y^{-\frac{1}{2}} dy = \int -2dt$

$2y^{\frac{1}{2}} = -2t + C$

$y^{\frac{1}{2}} = -t + C$

$y = (-t+C)^2$

$y(0) = (-0+C)^2 = 16 \rightarrow C = 4$

$y(t) = (-t+4)^2 = (4-t)^2$

The solution reaches zero at $t=4$

If $\frac{dy}{dt} = -2y$ $\quad \int \frac{dy}{y} = \int -2dt \quad \ln|y| = -2t + C \quad y = Ce^{-2t}$

$y(0) = Ce^{-2(0)} = 16 \quad C = 16 \quad y(t) = 16e^{-2t}$

at $t=4$ the depth would be $y(4) = 16e^{-2(4)} \approx 0.0054$

28. $\frac{db}{dt} = \lambda b - h = 0.5b - 1000$

ide. $u = 0.5b - 1000$ $\quad \frac{du}{dt} = 0.5$ $\quad 2du = dt$

$\int \frac{db}{0.5b - 1000} = \int dt$

$2\ln|0.5b - 1000| = t + C$

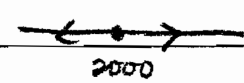
$\ln|0.5b - 1000| = \frac{1}{2}t + C$

$0.5b - 1000 = Ce^{\frac{1}{2}t}$

$b = \frac{Ce^{\frac{1}{2}t} + 1000}{0.5}$

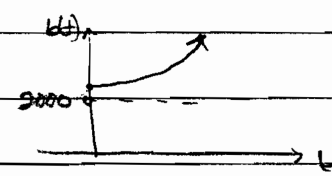
$b(t) = Ce^{\frac{1}{2}t} + 2000$

Recall from 5.2#24 eq pt at $b = 2000$



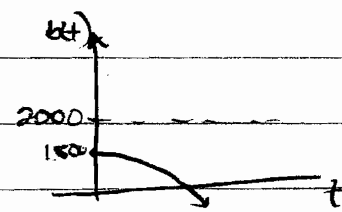
let $b(0) = 2001$

$b(0) = Ce^{\frac{1}{2}(0)} + 2000 = 2001 \Rightarrow C = 1 \qquad b(t) = e^{\frac{1}{2}t} + 2000$



let $b(0) = 1500$

$b(0) = Ce^{\frac{1}{2}(0)} + 2000 = 1500 \Rightarrow C = -500 \quad b(t) = -500e^{\frac{1}{2}t} + 2000$



Solutions match the phase line diagram