Math 1290 . HW9 Sec 1.9: 1, 6, 12, 14, 18, 22, 25, 26, 28, 36

1. 1L @ 30°C mixed with 2L @ 100°C

total volume = $1+2=3L$

$\therefore \frac{1}{3}$ is @ 30°C & $\frac{2}{3}$ is @ 100°C Temp $= \frac{1}{3}(30)+\frac{2}{3}(100) \approx \boxed{76.7°C}$

6. V_1 L @ 30°C mixed with V_2 L @ 100°

Total volume : $V_T = V_1 + V_2$

Temp $= \frac{V}{V_1+V_2}(30) + \frac{V_2}{V_1+V_2}(100)$

let $V_1 = 1$ L & $V_2 = 2L$

Temp $= \frac{1}{1+2}(30) + \frac{2}{1+2}(100) \approx \boxed{76.7°C}$ (same as #1)

12. 100 total

#	score	adjusted score
10	20	60
20	40	70
30	60	80
40	80	90

old average $= \frac{10}{100}(20) + \frac{20}{100}(40) + \frac{30}{100}(60) + \frac{40}{100}(80)$

$= 0.1(20)+0.2(40)+0.3(60)+0.4(80) = \boxed{60}$

new average $= 0.1(60)+0.2(70)+0.3(80)+0.4(90) = \boxed{80}$

Yes, the new average is the old average moved halfway to 100

14 $V = 1L$ $W = 0.1L$ $\gamma = 8 \frac{mmols}{L}$ $c_0 = 4 \frac{mmols}{L}$

a) $V \cdot c_0 = 1 \cdot 4 = 4$ mmols

b) $W \cdot c_0 = 0.1 \cdot 4 = 0.4$ mmols

c) $(V-W)c_0 = (1-0.1)(4) = (0.9)(4) = 3.6$ mmols

d) $W\gamma = 0.1(8) = 0.8$ mmols

e) $(V-W)c_0 + W\gamma = 3.6 + 0.8 = 4.4$

f) $\frac{4.4}{1} = 4.4 \frac{mmols}{L}$

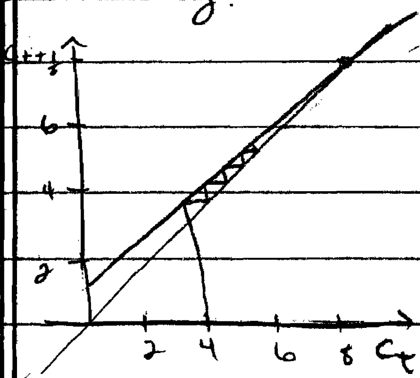
g) $c_{t+1} = (1-q)c_t + q\gamma = \left(1-\frac{W}{V}\right)c_t + \frac{W}{V}\gamma$

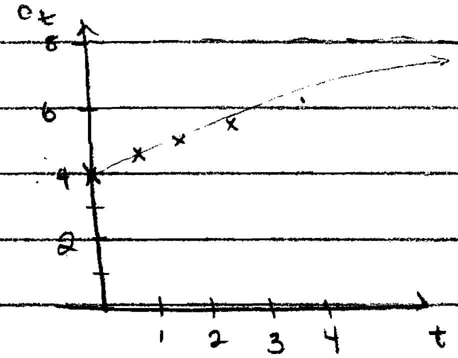
$c_{t+1} = \left(1-\frac{0.1}{1}\right)c_t + \frac{0.1}{1}(8) = 0.9c_t + 0.8$

$c_0 = 4$ $c_1 = 0.9(4) + 0.8 = 4.4$

18 from 14g. $C_{t+1} = 0.9C_t + 0.8$

t	C_t
0	4
1	4.4
2	4.76
3	5.084





notice the solution has $C_t = 8$ as a horizontal asymptote
AND
8 is the fixed point.

22 $C_{t+1} = (1-q)C_t + q\gamma$ where $q = \frac{w}{v} = \frac{0.1}{1} = 0.1$

$C_{t+1} = (1-0.1)C_t + 0.1(8)$ $\quad C_0 = 4.0 \frac{mmol}{L}$

$C_{t+1} = 0.9C_t + 0.8$ $\quad \gamma = 8$

Let $C^* = C_{t+1} = C_t$

$C^* = 0.9C^* + 0.8$

$0.1C^* = 0.8 \Rightarrow C^* = \frac{0.8}{0.1} = 8$

25 $\gamma = 0.21 \quad q = 0.4 \quad \alpha = 0.1$

method 1: $C_{t+1} = (1-q)(1-\alpha)C_t + q\gamma$

$C_{t+1} = (1-0.4)(1-0.1)C_t + 0.4(0.21)$

DDS: $C_{t+1} = 0.54C_t + 0.084$

Find eq pt let $C^* = C_{t+1} = C_t$

$C^* = 0.54C^* + 0.084$

$0.46C^* = 0.084$

$C^* = 0.1826$

method 2: use formula on pg 108

$C^* = \frac{q\gamma}{1-(1-q)(1-\alpha)}$

$C^* = \frac{(0.4)(0.21)}{1-(1-0.4)(1-0.1)}$

$C^* = \frac{0.84}{1-(0.6)(0.9)} = 0.1826$

The equilibrium concentration is higher $(0.1826 > 0.095)$ because more air exchanged w the the outside.

Here $q = 0.4$ vs $q = 0.2$ in example

26. $g = 0.1 \quad \alpha = 0.05 \quad \gamma = 0.21$
(again both methods from #25 are possible)
Method 1:

$C_{t+1} = 0.9(0.95)C_t + 0.021$
$C_{t+1} = 0.855\,C_t + 0.021$
Let $c^* = C_t = C_{t+1}$
$c^* = 0.855c^* + 0.021$
$0.145C^* = 0.021$
$C^* = \frac{0.021}{0.145} = 0.145$

The equilibrium concentration is about the same because $\frac{1}{2}$ as much is being absorbed ($\alpha = 0.05$) out of half as much being breathed in ($g = 0.1$)

No, gasping gets less oxygen because the fraction absorbed is much lower. Also, it takes much longer to reach equilibrium in this case.

28. $\gamma = 0.21 \quad g = 0.2 \quad C_t$ is reduced by 3% $\Rightarrow 0.03$

$C_{t+1} = (1-g)(C_t - 0.03) + g\gamma$
$C_{t+1} = (0.8)(C_t - 0.03) + 0.2(0.21)$
$C_{t+1} = 0.8C_t - 0.024 + 0.042$
$C_{t+1} = 0.8C_t + 0.018$
let $c^* = C_t = C_{t+1}$ $\quad c^* = 0.8c^* + 0.018$
$0.2c^* = 0.018$
$c^* = 0.09$

When C_t falls below 0.03 there is not enough for absorption, So, the function does not make sense for $C_t < 0.03$

36. $r = 0.2$ addition of 5×10^6 after each generation
a) $P_0 = 3 \times 10^6 \qquad P_1 = 0.2(3 \times 10^6) = 0.6 \times 10^6$ bacteria
b) $0.6 \times 10^6 + 5 \times 10^6 = 5.6 \times 10^6$ bacteria
c) $P_{t+1} = 0.2P_t + 5.0 \times 10^6$

Math 1290 HW10 Sec 1.10 2, 4, 10, 11, 12, 16, 33-36-38

2 $B_0 = 800 \quad R_0 = 200$

$B_{t+1} = 2(800) \quad R_{t+1} = 1(200)$

$B_1 = 1600 \qquad R_1 = 200$

fraction of $B_t = \frac{1600}{1600+200} = \frac{1600}{1800} = \frac{8}{9}$ Check: $\frac{1}{9} + \frac{8}{9} = \frac{9}{9} = 1$ ✓

fraction of $R_t = \frac{200}{1600+200} = \frac{200}{1800} = \frac{1}{9}$

4. $B_{t+1} = sB_t = 800s \qquad R_{t+1} = R_t = 200$

fraction of $B_t = \frac{800s}{800s+200}$ fraction of $R_t = \frac{200}{800s+200}$

check: $\frac{800s}{800s+200} + \frac{200}{800s+200} = \frac{800s+200}{800s+200} = 1$ ✓

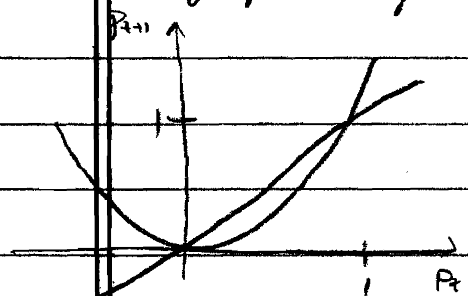
10 $p_{t+1} = \frac{sp_t}{sp_t + r(1-p_t)} \qquad s = 1.8 \quad r = 1.8$ $\quad m_t = 1.2\times10^5$, $b_t = 3.5\times10^6$

$p_t = \frac{1.2\times10^5}{1.2\times10^5 + 3.5\times10^6} = 0.03$ $\quad m_{t+1} = 1.8(1.2\times10^5) = 2.16\times10^5$, $b_{t+1} = 1.8(3.5\times10^6) = 6.3\times10^6$

$p_{t+1} = \frac{m_{t+1}}{m_{t+1}+b_{t+1}} = \frac{2.16\times10^5}{2.16\times10^5 + 3.5\times10^6} = 0.03$ no change in the fraction because both types are reproducing at the same rate $(r=s)$

11 $p_{t+1} = \frac{p_t}{p_t + 2(1-p_t)}$

method 1: graphically



fixed points at

$\boxed{p^* = 0 \ \& \ p^* = 1}$

method 2: algebraically

let $p^* = p_t = p_{t+1}$

$p^* = \frac{p^*}{p^* + 2(1-p^*)}$

$p^*(p^* + 2(1-p^*)) = p^*$

$(p^*)^2 + 2p^* - 2(p^*)^2 = p^*$

$-(p^*)^2 + p^* = 0$

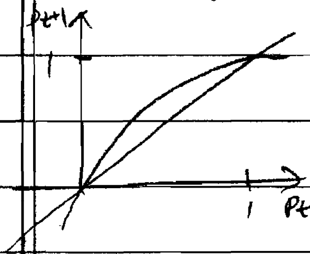
$p^*(-p^* + 1) = 0$

$\boxed{p^* = 0} \qquad -p^* + 1 = 0$

$\boxed{p^* = 1}$

12. $p_{t+1} = \dfrac{4p_t}{4p_t + 0.5(1-p_t)}$

method 1: Graphically



fixed points at $p^*=0$ & $p^*=1$

method 2: Algebraically let $p^* = p_t = p_{t+1}$

$$p^* = \frac{4p^*}{4p^* + 0.5(1-p^*)}$$

$$p^*(4p^* + 0.5 - 0.5p^*) = 4p^*$$

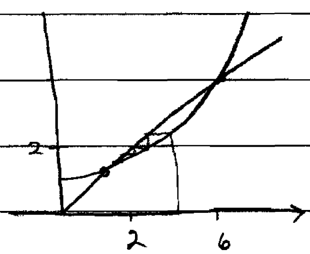
$$4(p^*)^2 + 0.5p^* - 0.5(p^*)^2 = 4p^*$$

$$3.5(p^*)^2 - 3.5p^* = 0$$

$$3.5p^*(p^*-1) = 0 \quad \Rightarrow p^* = 0 \text{ \& } p^* = 1$$

Important: Although the set up in #11 & 12 are very different having the same two fixed points indicates that there are only two possible longterm outcomes. Either the mutant population declines to zero or it takes over.

16.



The fixed point at ≈ 1.4 is stable (attracting) and the fixed point at ≈ 6.1 is unstable (repelling).

33.

$$\begin{array}{r} x_t = 100 \\ -20 \\ \hline 80 \\ +30 \\ \hline x_{t+1} = 110 \end{array} \qquad \begin{array}{r} y_t = 100 \\ -30 \\ \hline 70 \\ +20 \\ \hline y_{t+1} = 90 \end{array} \qquad \begin{array}{r} x_{t+1} = 110 \\ -22 \\ \hline 88 \\ +27 \\ \hline x_{t+2} = 115 \end{array} \qquad \begin{array}{r} y_{t+1} = 90 \\ -27 \\ \hline 63 \\ +22 \\ \hline y_{t+2} = 85 \end{array}$$

34.

$$\begin{array}{r} x_t = 200 \\ -40 \\ \hline x_{t+1} = 160 \end{array} \qquad \begin{array}{r} y_t = 0 \\ +40 \\ \hline y_{t+1} = 40 \end{array} \qquad \begin{array}{r} x_{t+1} = 160 \\ -32 \\ \hline 128 \\ +12 \\ \hline x_{t+2} = 140 \end{array} \qquad \begin{array}{r} y_{t+1} = 40 \\ -12 \\ \hline 28 \\ +32 \\ \hline y_{t+2} = 60 \end{array}$$

35.

$$x_{t+1} = x_t - 0.2x_t + 0.3y_t = 0.8x_t + 0.3y_t$$

$$y_{t+1} = y_t - 0.3y_t + 0.2x_t = 0.2x_t + 0.7y_t$$

36a) Because no butterflies dying or reproducing the total population remains the same for all time: $x_{t+1} + y_{t+1} = x_t + y_t$

$$p_{t+1} = \frac{x_{t+1}}{x_{t+1}+y_{t+1}} = \frac{0.8x_t + 0.3y_t}{x_t + y_t} = \frac{0.8\,x_t}{x_t+y_t} + \frac{0.3y_t}{x_t+y_t}$$

let $p_t = \frac{x_t}{x_t+y_t}$ Then $1-p_t = \frac{y_t}{x_t+y_t}$ $\Longrightarrow$ $\boxed{p_{t+1} = 0.8p_t + 0.3(1-p_t)}$

Find equilibrium: let $p^* = p_t = p_{t+1}$

$$p^* = 0.8p^* + 0.3(1-p^*)$$
$$p^* = 0.8p^* + 0.3 - 0.3p^*$$
$$0.5p^* = 0.3$$
$$p^* = \frac{0.3}{0.5} = \boxed{0.6}$$

38 a)

$x_t = 100$	$y_t = 100$	$x_{t+1} = 190$	$y_{t+1} = 160$	
$-\ 20$	-30	$-\ 38$	$-\ 48$	
80	70	152	112	
$+130$	$+20$	$+\ 48$	$+\ 38$	
Before migration → $\boxed{110}$	$\boxed{90}$	200	150	← after migration
80	$+\ 70$	$+\ 152$	$+112$	
after migration → $\boxed{190}$ x_{t+1}	$\boxed{160}$ y_{t+1}	$x_{t+2} = 352$	$y_{t+2} = 262$	← after reproduction

b) $x_{t+1} = 2(x_t - 0.2x_t) + 0.3y_t$ $\qquad y_{t+1} = 2(y_t - 0.3y_t) + 0.2x_t$

$x_{t+1} = 1.6x_t + 0.3y_t$ $\qquad y_{t+1} = 1.4y_t + 0.2x_t$

c)
$$p_{t+1} = \frac{1.6x_t + 0.3y_t}{1.8x_t + 1.7y_t} = \frac{\frac{1.6x_t}{x_t+y_t} + \frac{0.3y_t}{x_t+y_t}}{\frac{1.8x_t}{x_t+y_t} + \frac{1.7y_t}{x_t+y_t}} = \frac{1.6p_t + 0.3(1-p_t)}{1.8p_t + 1.7(1-p_t)}$$

d) let $p^* = p_t = p_{t+1}$

$$p^* = \frac{1.6p^* + 0.3(1-p^*)}{1.8p^* + 1.7(1-p^*)} = \frac{1.6p^* + 0.3 - 0.3p^*}{1.8p^* + 1.7 - 1.7p^*} = \frac{1.3p^* + 0.3}{0.1p^* + 1.7}$$

$$p^*(0.1p^* + 1.7) = 1.3p^* + 0.3$$
$$0.1(p^*)^2 + 1.7p^* = 1.3p^* + 0.3$$
$$0.1(p^*)^2 + 0.4p^* - 0.3 = 0$$

use quadratic w/ $a = 0.1$ $b = 0.4$ $c = -0.3$

$$p^* = \frac{-(0.4) \pm \sqrt{(0.4)^2 - 4(0.1)(-0.3)}}{2(0.1)}$$

$p^* \approx 0.646$ & ~~$p^* \approx -4.645$~~

e)