Math 1297 Test 2 practice problems. B. Peckham. March 23, 2007

1. Determine the following limits (if they exist):

   (a) $\lim_{x\to 0} \frac{e^x - x - 1}{x^2}$

   (b) $\lim_{x\to\infty} (\frac{x}{x+1})^x$

2. How many integrations by parts would be necessary to find $\int x^{10}\cos(3x)dx$? Explain briefly.

3. Do the following integral: $\int 3x\cos(2x)dx$.

4. Integrate $\int_0^{\pi/2} \sin^3(x)\cos(x)dx$ in two ways (using two different u substitutions).

5. Integrate $\int \cos(2x)\sin(5x)dx$ using the trig identitiy: $\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$.

6. Consider the following integral: $\int \frac{x^3}{\sqrt{4+x^2}}dx$. Make the substitution $x = 2\tan(\theta)$ to rewrite the integral with respect to θ. Simplify enough so that the integrand has no square roots in it, but DO NOT EVALUATE THE INTEGRAL.

7. Write the form of the partial fractional decomposition for each of the following. DO NOT SOLVE FOR THE CONSTANTS.

   (a) $\frac{1+x^2}{x(x^2+9)}$.

   (b) $\frac{2x^3 - 3x + 1}{x^3(x-2)}$.

8. Use the fact that $\frac{3x^2+4x-2}{x(x+1)(2x-1)} = \frac{2}{x} - \frac{1}{x+1} + \frac{1}{2x-1}$ to find $\int \frac{3x^2+4x-2}{x(x+1)(2x-1)}dx$.

9. Evaluate $\int \frac{6x+1}{3x+2}dx$

10. Consider the integral $\int \frac{\sqrt{x}}{x-4}dx$. Determine the integral with respect to u after making the substitution $u = \sqrt{x}$. Is the new integral any easier to integrate than the original one? Explain why briefly (but DON'T EVALUATE).

11. (5pts) The following integral comes from a table of integrals:

$$\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2 u} + C.$$

    Use it to determine

$$\int \frac{dx}{x^2\sqrt{4x^2-3}}$$

12. Order the following estimates for $\int_0^3 x^2 dx$: L_3, R_3, M_3, T_3. (The notation stands for estimates obtained by using 3 equal length intervals and the Left endpoint, Right endpoint, Midpoint, and Trapezoidal methods, respectively.) Sketch a picture to illustate L_3.

13. Evaluate $\int_0^1 \frac{1}{\sqrt{x}} dx$ using the formal definition of the improper integral.

14. Set up an integral with respect to x for the arclength along the curve $y = 1/x$ from $(1, 1)$ to $(2, 1/2)$ Set up an integral with respect to y to compute the same arclength. DO NOT EVALUATE EITHER INTEGRAL.

15. Set up an integral to evaluate the surface area of the surface of revolution obtained by revolving the parabola $y = x^2$ from the point $(0, 0)$ to $(3, 9)$ around the y axis. DO NOT EVALUATE THE INTEGRAL.

16. True or False? If $\sum_{n=1}^{\infty} a_n$ converges then $\{a_n\}_{n=1}^{\infty}$ must converge. Explain briefly.

17. Give an example of a sequence that is bounded but not monotonic.

18. What is $\lim_{n\to\infty} \frac{2n + 500}{2n^2 + 1}$? Indicate your reasoning briefly.

19. Consider the following list of series.

    (a) $\sum_{n=2}^{\infty} \frac{2^{n+1}}{3^{n-1}}$

    (b) $-1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \cdots$

    (c) $\sum_{n=1}^{\infty} \frac{n^2 + 5n}{2n^2 + 3}$

    - Which of the above series are geometric?
    - What is "r" for the series that are geometric?
    - Which of the above series converge? Justify briefly. No formal proof required.
    - To what number does each geometric series converge?

20. Prove directly (by obtaining a formula for the partial sums s_n) that $\sum_{n=1}^{\infty} \frac{1}{3^{n-1}} = \frac{3}{2}$.