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Diff. Equations and Lin. Alg.
Math 3280
Quiz 1, Fall 2018
B. Peckham

1. (3 pts) For what value(s) of $A$ is the function $\phi(x) = Ae^{5x}$ a solution to

$$y'' - 3y' + y = 3e^{5x}?$$

$\phi'(x) = 5Ae^{5x}$

$\phi''(x) = 25Ae^{5x}$

Show your work.

Plug in: $(25Ae^{5x}) - 3(5Ae^{5x}) + Ae^{5x} = 3e^{5x}$

i.e., $25A - 15A + A = 3$

$11A = 3 \Rightarrow A = \frac{3}{11} \Rightarrow \frac{3}{11}e^{5x}$ is a sol.

2. Consider the differential equation $\frac{dy}{dt} = y^2t^2$

(a) (3 pts) Find the general solution.

$$\int \frac{1}{y^2}\frac{dy}{dt}\,dt = \int t^2\,dt + C$$

$$-\frac{1}{y} = \frac{t^3}{3} + C \Rightarrow y = \frac{-1}{\frac{t^3}{3} + C}$$

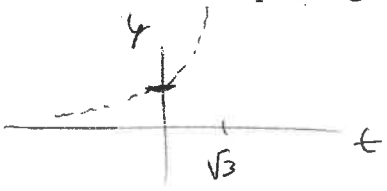
(b) (2pts) What is the solution corresponding to $y(0) = 1$?

$y(0) = 1 \Rightarrow \frac{-1}{\frac{0^3}{3} + C} = 1 \Rightarrow \frac{-1}{C} = 1 \Rightarrow C = -1$

$\therefore y(t) = \frac{-1}{\frac{t^3}{3} - 1} = \frac{1}{1 - \frac{t^3}{3}} = \frac{3}{3 - t^3}$

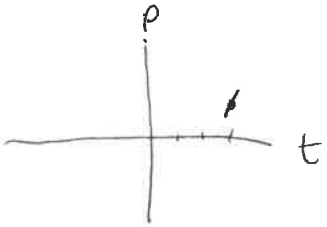
(c) (+2 pts Extra Credtit) What is the domain of the solution corresponding to the initial condition $y(0) = 1$?

Problem $t^3 = 3$ i.e., $t = \sqrt[3]{3}$




$\Rightarrow D = (-\infty, \sqrt[3]{3})$

3. (2pts) If $P(t)$ is a solution to $\frac{dP}{dt} = P^2 + 2t$, and $P(t)$ satisfies $P(3) = 1$, what is $P'(3)$? (Hint: this is a slope field question.)




$P'(3) = P(3)^2 + 2 \cdot 3$

$= 1^2 + 6 = 7$