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Diff. Equations and Lin. Alg.
Math 3280
Quiz 3, Fall 2018
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Consider the following initial value problem.

$$\frac{dP(t)}{dt} = -2P(t), \quad P(0) = 2$$

1. Find the general solution to the differential equation without the initial condition

   (a) (~~1~~ pts) By inspection. $P(t) = P_0 e^{-2t}$ — E.C. Explicit sln.

   (b) (2 pts) Using separation of variables. for an implicit sln.

   $\frac{1}{P}\frac{dP}{dt} = -2 \Rightarrow \int \frac{1}{P}\frac{dP}{dt}\,dt = \int -2\,dt$

   $\Rightarrow \ln|P| = -2t + C \Rightarrow |P| = e^{C} \cdot e^{-2t}$

   Case 1: $P>0 \Rightarrow P(t) = ke^{-2t}$ w/ $k>0$ (Since $k = e^{c} > 0$)

   Case 2: $P<0 \Rightarrow P(t) = -e^{c}e^{-2t} = ke^{-2t}$ w/ $k<0$

   Case 3: $P=0 \Rightarrow 0e^{-2t} = ke^{-2t}$ w/ $k=0$.

   Combine Cases: $P(t) = ke^{-2t}$ w/ k any real #.

   (c) (2 pts) Using the first order linear techniques with an integrating factor

   Rewrite $\dot{P} + 2P = 0$.

   int. factor: $e^{\int 2\,dt} = e^{2t}$

   $\Rightarrow e^{2t}\dot{P} + 2Pe^{2t} = 0$

   $\Rightarrow (e^{2t}P)' = 0$

   integrate: $\int (e^{2t}P)'\,dt = \int 0\,dt + C$

   i.e. $e^{2t}P = 0 + C$

   $\Rightarrow P(t) = Ce^{-2t}$

2. (1 pt) Use the general solution from any one of a-c and the initial condition $P(0) = 2$ to find the soution to the initial value problem.

   $P(t) = Ce^{-2t}$ and $P(0) = 2 \Rightarrow Ce^{-2 \cdot 0} = 2 \Rightarrow C = 2$

   $\therefore P(t) = 2e^{-2t}$

3. (2 pts) Use Euler's method to numerically estimate the $P(1.0)$ assuming $P(t)$ is the solution to the differential equation which satisfies the initial condition $P(0) = 2$. Use a time step of $h = \cancel{0.5}\ .25$

n	t_n	P_n	$f(t_n,P_n) = -2P_n$
0	0	2	-4
1	.25	1	
2	0.5	0.5	
3	0.75	0.25	
4	1.0	0.125	

   $P_{n+1} = P_n + h \cdot f(t_n, P_n) = P_n + (.25)\cdot(-2P_n) = P_n/2$

   $P_1 = 2 + (.25)(-2)\,2 = 1$

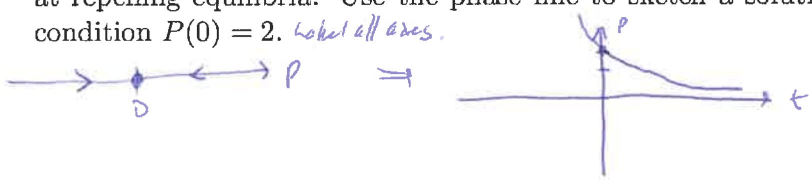
   $P_2 = 1 + .25\,(-1)\cdot 2 = .5$

   $P_3 = 0.5 + (.25)(-.5)\cdot 2 = .25$

   $P_4 = .25 - \frac{.25}{2} = 0.125$

   So $\boxed{P(1) \approx 0.125}$

4. (2 pts) Sketch and label a phase line, with solid dots at attracting equilibria and open dots at repelling equilibria. Use the phase line to sketch a solution corresponding to the initial condition $P(0) = 2$. Label all axes.



5. (2pts EC) Sketch a slope field including slope marks along the lines $P = 2, P = 1, P = 0.5, P = -1$. Sketch the solution on this slope field corresponding to $P(0) = 2$. Extend your solution both forward and backward in time. Label your axes.