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Diff. Equations and Lin. Alg.
Math 3280
Quiz 4, Fall 2018
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1. (3 pts) Use USING GAUSSIAN ELIMINATION (row reduction operations) on the "augmented matrix" to convert it to row echelon form to find all solutions to the following system. Write your answer in vector form.

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}.$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 1 & 2 & 1 & 1 \end{array}\right] \xrightarrow{R2-R1} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 0 & 2 & -1 & -3 \end{array}\right] \text{ (r.e.f.)} \Rightarrow$$

$x_3 = t$ (free)

$2x_2 - x_3 = -3$

$\Rightarrow 2x_2 = -3 + x_3 = -3 + t$

$x_1 + 0x_2 + 2x_3 = 4$

$\Rightarrow x_1 = 4 - 2x_3 = 4 - 2t$

$$\therefore \vec{x} = \begin{bmatrix} 4-2t \\ \frac{-3}{2}+\frac{t}{2} \\ t \end{bmatrix} = \begin{bmatrix} 4 \\ -\frac{3}{2} \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ \frac{1}{2} \\ 1 \end{bmatrix}$$

2. Consider the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

(a) (2pts) Compute $\det(A)$. Show your work.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\det A = 1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - 0\begin{vmatrix} & \\ & \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1\cdot 1 - 0 + 2 \cdot 1 = 3$$

$\frac{1}{2} - \frac{1}{4} = \frac{9}{16} - \frac{3}{16}$

(b) (3pts) Compute A^{-1}. Hint: use row reduction.

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{3}{2} & \frac{1}{2} & -\frac{1}{2} & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & -\frac{4}{3} \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{array}\right]$$

A^{-1}

(c) (2pts) Use your answer for (b) to solve $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Do not solve using row reduction (other than computing A^{-1} in (b).)

$$\vec{x} = A^{-1}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 2 & -4 \\ -1 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$$

3. (2 pts extra credit) Write down a system of equations that has 5 unknowns and whose solution has 3 free variables. Need 2 eqns.

For example

$1x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 = 0$
$0x_1 + x_2 + 0x_3 + 0x_4 + 0x_5 = 0$

(x_3, x_4, x_5 are free)

Lots of other correct answers.