Name AK

Diff. Equations and Lin. Alg.
Math 3280, B. Peckham
Quiz 5, Fall 2018

1. (3 pts) Determine whether $\vec{w}$ is a linear combination of $\vec{v}_1$ and $\vec{v}_2$, where $\vec{w} = \begin{bmatrix}1\\0\\7\end{bmatrix}$, $\vec{v}_1 = \begin{bmatrix}1\\3\\4\end{bmatrix}$, and $\vec{v}_2 = \begin{bmatrix}3\\2\\5\end{bmatrix}$. Justify fully.

Solve $c_1\vec{v}_1 + c_2\vec{v}_2 = \vec{w}$.

ie $c_1\begin{bmatrix}1\\3\\4\end{bmatrix} + c_2\begin{bmatrix}3\\2\\5\end{bmatrix} = \begin{bmatrix}1\\0\\7\end{bmatrix}$

ie $\begin{bmatrix}1 & 3\\3 & 2\\4 & 5\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}1\\0\\7\end{bmatrix}$

rowred: $\left[\begin{array}{cc|c}1 & 3 & 1\\3 & 2 & 0\\4 & 5 & 7\end{array}\right] \to \left[\begin{array}{cc|c}1 & 3 & 1\\0 & -7 & -3\\0 & -7 & 3\end{array}\right] \to \left[\begin{array}{cc|c}1 & 3 & 1\\0 & -7 & -3\\0 & 0 & 6\end{array}\right] \Rightarrow$ no sln. $\Rightarrow$ $\vec{w}$ is not a l.c. of $\vec{v}_1$ and $\vec{v}_2$

2. Let $S = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\right\}$. Show directly from the definition of linearly independent and span (not just computing a determinant) that

(a) (2pts) S is a linearly independent set

Solve $c_1\begin{bmatrix}1\\0\end{bmatrix} + c_2\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$ ie $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$ Already reduced:

$\left[\begin{array}{cc|c}1 & 1 & 0\\0 & 1 & 0\end{array}\right] \Rightarrow c_2 = 0$, $c_1 + c_2 = 0 \Rightarrow c_1 = 0$ $\therefore c_1 = 0 = c_2$

$\therefore S$ is a lin indep. set.

(b) (2pts) the span of S is all of $\Re^2$.

Solve: $c_1\begin{bmatrix}1\\0\end{bmatrix} + c_2\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}x\\y\end{bmatrix}$.

$\Rightarrow$ rowreduce: $\left[\begin{array}{cc|c}1 & 1 & x\\0 & 1 & y\end{array}\right] \Rightarrow c_2 = y$, $c_1 + c_2 = x$, ie $c_1 = x - c_2 = x - y$

$\therefore \begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}x-y\\y\end{bmatrix}$. Since there is a $\begin{bmatrix}c_1\\c_2\end{bmatrix}$ for any $\begin{bmatrix}x\\y\end{bmatrix}$ in $\mathbb{R}^2$, then Span S is all of $\mathbb{R}^2$

3. (3 pts) Find a basis for the set of solutions to the vector equation

$$\begin{bmatrix}1 & 2 & 3 & 0\\0 & 2 & 4 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}.$$

Rows reduce: (nothing to do)

Interpret: leading var's: x_1, x_2

free var's: x_3, x_4

$\therefore$ let $x_3 = s$, $x_4 = t$

2nd row $\Rightarrow$ $2x_2 + 4x_3 + 0x_4 = 0 \Rightarrow x_2 = -\frac{4}{2}x_3 = -2s$

top row $\Rightarrow$ $1x_1 + 2x_2 + 3x_3 + 0x_4 = 0 \Rightarrow x_1 = -2x_2 - 3x_3 = -2(-2s) - 3s = s$

Slns: $s\begin{bmatrix}1\\-2\\1\\0\end{bmatrix} + t\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

So $\left\{\begin{bmatrix}1\\-2\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right\}$ is a basis for the set of solutions.

4. (2 pts EC) Let $W = \{(a, b, c, c) \in \Re^4 : a + b + c = 1\}$. Define a matrix A, and a vector $\vec{b}$ such that $W = \{\vec{x} \in \Re^4 : A\vec{x} = \vec{b}\}$.

$(a, b, c, c) \Rightarrow x_3 = x_4$ or ~~$0x_1+0x_2+\ldots$~~

$0x_1 + 0x_2 + 1x_3 - 1x_4 = 0$

$a + b + c = 1 \leftrightarrow x_1 + x_2 + x_3 + 0x_4 = 1$

$\therefore A = \begin{bmatrix}0 & 0 & 1 & -1\\1 & 1 & 1 & 0\end{bmatrix}$, $\vec{b} = \begin{bmatrix}0\\1\end{bmatrix}$