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Diff. Equations and Lin. Alg.
Math 3280, B. Peckham
Quiz 6, Fall 2018

1. (3pts) Find a basis for the subspace of $\Re^3$ determined by $\{\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \Re^3 : x_1 = x_3\}$. (You need not prove your answer is a basis.)

Solve $\cancel{1}x_1 + 0x_2 + -1x_3 = 0$

$x_3 = t,\ x_2 = s$

$x_1 = x_3 = t$

$\therefore \vec{x} = \begin{bmatrix} t \\ s \\ t \end{bmatrix} = t\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\Rightarrow \beta = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

2. (3pts) Determine whether the following set of functions (defined on $\Re$) is linearly independent or dependent: $\{e^x, e^{2x}, xe^x\}$. Work directly from the definition of linear independence, not just a determinant. Note: Method 2 is easier.

Solve $c_1 e^x + c_2 e^{2x} + c_3 xe^x = 0$ (for all x)

Method 1: True for all $x \Rightarrow$ true for $x = -1, 0, 1$ (or any other 3 values of x)

$\Rightarrow$ $x=-1$: $c_1 e^{-1} + c_2 e^{-2} + c_3(-1)e^{-1} = 0$
$x=0$: $c_1 + c_2 + 0 = 0$
$x=1$: $c_1 e^1 + c_2 e^2 + c_3 2e^2 = 0$

Method 2: diff 2 times:

$c_1 e^x + c_2 e^{2x} + c_3 xe^x = 0$
$c_1 e^x + 2c_2 e^{2x} + c_3(xe^x + e^x) = 0$
$c_1 e^x + 4c_2 e^{2x} + c_3(xe^x + 2e^x) = 0$

True for all $x \Rightarrow$ true for $x=0$:

$1c_1 + 1c_2 + 0c_3 = 0$
$1c_1 + 2c_2 + 1c_3 = 0$
$1c_1 + 4c_2 + 2c_3 = 0$

Row reduce ⋮ missing steps ⋮ $\Rightarrow c_1 = c_2 = c_3 = 0$ $\therefore$ independent

row reduction:

$\left[\begin{array}{ccc|c} e^{-1} & e^{-2} & -e^{-1} & 0 \\ 1 & 1 & 0 & 0 \\ e^1 & e^2 & 2e^2 & 0 \end{array}\right] \xrightarrow[R2/e]{(R1)\cdot e} \left[\begin{array}{ccc|c} 1 & e^{-1} & -1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & e & 2e & 0 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|c} 1 & e^{-1} & -1 & 0 \\ 0 & 1-e^{-1} & 1 & 0 \\ 0 & e-e^{-1} & 2e+1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cccc} 1 & e^{-1} & -1 & 0 \\ 0 & 1 & \frac{1}{1-e^{-1}} & 0 \\ 0 & 0 & * & 0 \end{array}\right]$

where $* \neq 0$.

$\Rightarrow c_1 = c_2 = c_3 = 0$.

$\therefore$ lin indep

3. (3pts) Find the general solution to $y'' + 2y' - 15y = 0$. Hint: try functions of the form e^{rx}.

Try $y = e^{rx} \Rightarrow y' = re^{rx}$, $y'' = r^2 e^{rx}$. Plug in: $r^2 e^{rx} + 2re^{rx} + (-15)e^{rx} = 0$

ie, $e^{rx}(r^2 + 2r - 15) = 0 \Rightarrow e^{rx}(r+5)(r-3) = 0 \Rightarrow r = -5, 3$

$\therefore$ Let $y_1(x) = e^{-5x}$, $y_2(x) = e^{3x}$

$\Rightarrow$ gen'l sln is $\underline{y(x) = c_1 e^{-5x} + c_2 e^{3x}}$

4. (1pt + 2pts EC) Consider the differential equation $ay'' + by' + cy = e^{4x}$. Assume a, b, c are real constants. Assume also that you know two functions that are solutions to this differential equation: y_1 and y_2. Show that the function y_3, defined by $y_3(x) = y_1(x) \mp y_2(x)$, is a solution to $ay'' + by' + cy = 0$.

y_1, y_2 slns $\Rightarrow ay_1'' + by_1' + cy_1 = e^{4x}$ and $ay_2'' + by_2' + cy_2 = e^{4x}$

$\therefore a(y_1 - y_2)'' + b(y_1 - y_2)' + c(y_1 - y_2) = (ay_1'' + by_1' + cy_1) - (ay_2'' + by_2' + cy_2)$
$= e^{4x} - e^{4x} = 0$

ie, $y_1 - y_2$ is a sln to $ay'' + by' + cy = 0$. //

2 Method 2 details of row reduction:

$$\left[\begin{array}{ccc:c} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 4 & 2 & 0 \end{array}\right] \xrightarrow[R3-R1]{R2-R1} \left[\begin{array}{ccc:c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 3 & 2 & 0 \end{array}\right] \xrightarrow{R3-3R2} \left[\begin{array}{ccc:c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right]$$

last row $\Rightarrow c_3 = 0$. Back solving $\Rightarrow c_2 = 0$ and $c_1 = 0$